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A HYBRID SMOOTHING-NONSMOOTH NEWTON-TYPE ALGORITHM YIELDING AN EXACT SOLUTION OF THE P_0-LCP *[1)]

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Abstract

We propose a hybrid smoothing-nonsmooth Newton-type algorithm for solving the P_0 linear complementarity problem (P_0-LCP) based on the techniques used in the non-smooth Newton method and smoothing Newton method. Under some assumptions, the proposed algorithm can find an exact solution of P_0-LCP in finite steps. Preliminary numerical results indicate that the proposed algorithm is promising.

Mathematics subject classification: 90C33, 65k10.
Key words: P_0 linear complementarity problem, Hybrid smoothing-nonsmooth Newton-type method, Finite termination.

1. Introduction

It is well-known that many mathematical programming problems can be reformulated as a non-smooth equation. By using general Jacobian in the sense of Clarke [4], one can treat directly the non-smooth equation and design a few iterative Newton-type algorithms to solve the problem. This is known as the *non-smooth Newton method* [6, 15]. Moreover, to overcome the difficulties arising from non-differentiability of the non-smooth equation, one can smooth the non-smooth equation by using some smoothing functions. Instead of the non-smooth equation, one investigates a system of the parameterized smoothing equations. Furthermore, one can design a few iterative Newton-type algorithms to solve the problem. This is just the *non-interior continuation method / smoothing Newton method*, which has been used extensively to solve a few mathematical programming problems [1, 2, 8, 16].

It is also known that the iterative method only generates generally an approximation solution of the problem concerned. In order to obtain an exact solution of the problem, many algorithms with the finite termination property have been proposed to solve some linear optimization problems including the linear programming [14, 18], the linear complementarity problem [6, 17], the box constrained variational inequality problem [3, 13], and the vertical linear complementarity problem [5]. It is shown in each above-mentioned algorithm that an exact solution of the problem can be found in one step when an iterate is sufficiently close to this solution.

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In this paper, we consider the P_0 linear complementarity problem (P_0-LCP) of finding a vector $(x, y) \in R^n \times R^n$ such that

$$x \geq 0, \quad y \geq 0, \quad x^T y = 0, \quad Mx + q - y = 0, \tag{1.1}$$

where the matrix $M \in R^{n\times n}$ is a P_0-matrix and the vector $q \in R^n$. By exploiting the techniques of non-smooth Newton methods [6] and smoothing Newton methods [9, 16], we give a hybrid smoothing-nonsmooth Newton-type algorithm for the P_0-LCP (1.1) where we use a smoothing function introduced by Huang-Han-Chen [9] in the smoothing Newton step. It is shown that our algorithm can find an exact solution of the P_0-LCP (1.1) in finite steps under some assumptions. We implement the proposed algorithm for several standard test problems by a MATLAB code. The preliminary numerical results indicate that the algorithm is promising.

The rest of this paper is organized as follows. We give some properties of the smoothing function introduced in [9] and some basic concepts in the next section. Then we propose a hybrid smoothing-nonsmooth Newton-type algorithm for the P_0-LCP (1.1). In section 3, we show the finite termination property of the proposed algorithm. Some numerical results are given in section 4.

The following notions will be used throughout this paper. All vector are column vectors, the subscript T denotes transpose, R^n (respectively, R) denotes the space of n-dimensional real column vectors (respectively, real numbers), R^n_+ and R^n_{++} denote the nonnegative and positive orthants of R^n, R_+ (respectively, R_{++}) denotes the nonnegative (respectively, positive) orthant in R. We define $N := \{1, 2, \cdots, n\}$. For any vector $u \in R^n$, we denote by $diag\{u_i : i \in N\}$ the diagonal matrix whose ith diagonal element is u_i and $vec\{u_i : i \in N\}$ the vector u. For simplicity, we use (u, v) for the column vector $(u^T, v^T)^T$. The matrix I represents the identity matrix of arbitrary dimension. The symbol $\|\cdot\|$ stands for the 2-norm. We denote by S the solution set of the P_0-LCP (1.1).

2. Algorithm Description

It is easy to see that the problem (1.1) is equivalent to the following non-smooth equations

$$F(w) := F(x, y) := \begin{pmatrix} y - Mx - q \\ \min\{x, y\} \end{pmatrix} = 0, \tag{2.1}$$

that is, $(x, y) \in S$ if and only if $F(w) = 0$. Since $F(w)$ is a locally Lipschitz-continuous operator, we can define its generalized Jacobian (see [6])

$$\partial F(w) = \{V_a | a_i = 1 \text{ if } x_i < y_i, \quad a_i = 0 \text{ if } x_i > y_i, \quad a_i \in [0, 1] \text{ if } x_i = y_i, \quad i \in N\},$$

where

$$V_a := \begin{pmatrix} -M & I \\ D_a & I - D_a \end{pmatrix}, \quad D_a := diag(a_1, \cdots, a_n).$$

Moreover, the non-smooth function $\min\{x, y\}$ in (2.1) can be smoothed by using the smoothing function $\phi : R^3 \to R$ defined by

$$\phi(\mu, a, b) = (1 + \mu)(a + b) - \sqrt{(1 - \mu)^2 (a - b)^2 + 4\mu^2}, \tag{2.2}$$

which was introduced by Huang-Han-Chen [9]. We need the following properties of smoothing function (2.2) which can be found in [9].

Lemma 2.1. *Let $(\mu, a, b) \in R^3$ and $\phi(\mu, a, b)$ be defined by (2.2). Then $\phi(\mu, a, b)$ is continuously differentiable in $R_{++} \times R^2$. Moreover, $\phi(0, a, b) = 0$ if and only if $a, b \geq 0, ab = 0$.*