Journal of Computational Mathematics, Vol.15, No.2, 1997, 149–158.

A CLASS OF C^1 DISCRETE INTERPOLANTS OVER TETRAHEDRA*

X.C. Liu
(*Institute of Image Processing and Pattern Recognition, Shanghai Jiao Tong University, Shanghai, China*)

Abstract

Smooth interpolants defined over tetrahedra are currently being developed for they have many applications in geography, solid modeling, finite element analysis, etc. In this paper, we will characterize a certain class of C^1 discrete tetrahedral interpolants with only C^1 data required. As special cases of the class characterized, we give two C^1 discrete tetrahedral interpolants which have concise expressions.

1. Introduction

The purpose of the paper is to characterize a certain class of C^1 discrete interpolants defined over tetrahedra with only C^1 data required. We assume that a polyhedral domain in three-space or a set of 3D scattered data have been tessellated into tetrahedra with any two of which share only one face. As for this preprocessing stage, one may refer to [2] and [3] and here we omit it. In the paper, we only describe the characterization of an interpolant over a single tetrahedron for the interpolants have the same form. Now we begin our paper with some conceptions and notations.

A discrete interpolant $\mathcal{P}$ is said to interpolate a linear functional $\mathcal{L}$ if $\mathcal{LP}f = f$ for any function f. For simplicity, we sometimes use $\mathcal{P}$ to denote $\mathcal{P}f$ for any function f being interpolated. Denote a general tetrahedron by V with vertices V_i, $i = 1, \cdots, 4$. Denote its faces by F_i, $i = 1, \cdots, 4$, with F_i opposite to vertex V_i, and edges by E_i^j, $j \neq i$, with E_i^j opposite to vertices V_j and V_i, i.e.,

$$E_i^j(t) = (1-t)V_k + tV_l, \quad k, l \neq i, j, \quad k \neq l.$$

Denote edge vectors by $e_{ij} = V_j - V_i$. Furthermore, denote(see Fig.1)

$$\boldsymbol{n}_i = e_{li} - \frac{\boldsymbol{n}_l^i \cdot e_{li}}{\boldsymbol{n}_l^i \cdot \boldsymbol{n}_l^i}\boldsymbol{n}_l^i, \quad \boldsymbol{n}_l^i = e_{kl} - \frac{e_{jk} \cdot e_{kl}}{e_{jk} \cdot e_{jk}} e_{jk} \tag{1.1}$$

where $(i, j, k, l) \in \Lambda := \{(1,2,3,4), (2,3,4,1), (3,4,1,2), (4,1,2,3)\}$, then the directional derivative is computed to be

$$\frac{\partial b_i}{\partial \boldsymbol{n}_i} = 1, \quad \frac{\partial b_j}{\partial \boldsymbol{n}_i} = -\frac{\boldsymbol{n}_l^i \cdot e_{li}}{\boldsymbol{n}_l^i \cdot \boldsymbol{n}_l^i} \cdot \frac{e_{jk} \cdot e_{kl}}{e_{jk} \cdot e_{jk}} \tag{1.2}$$

* Received January 19, 1995.

$$\frac{\partial b_k}{\partial \boldsymbol{n}_i} = -\frac{\boldsymbol{n}_l^i \cdot e_{li}}{\boldsymbol{n}_l^i \cdot \boldsymbol{n}_l^i} \cdot \frac{e_{jk} \cdot e_{lj}}{e_{jk} \cdot e_{jk}}, \quad \frac{\partial b_l}{\partial \boldsymbol{n}_i} = -1 - \frac{\boldsymbol{n}_l^i \cdot e_{li}}{\boldsymbol{n}_l^i \cdot \boldsymbol{n}_l^i}, \tag{1.3}$$

where (b_1, b_2, b_3, b_4) is the barycentric coordinate of a point P on tetrahedron V:

$$P = b_1V_1 + b_2V_2 + b_3V_3 + b_4V_4, \quad b_1 + b_2 + b_3 + b_4 = 1.$$

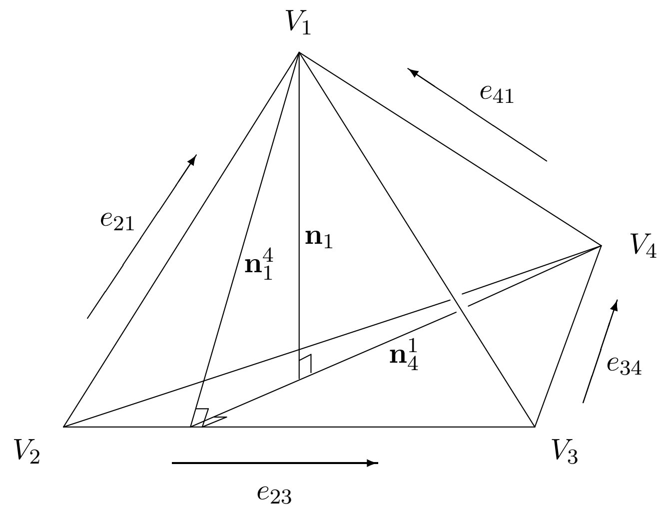

Fig. 1 Notational conventions on a tetrahedron

2. Interpolation Requirements

Our goal is to characterize the set of discrete tetrahedral interpolants $\mathcal{P}$ which satisfies the following requirements:

(2.1). $\mathcal{P}$ forms C^1 joins with adjacent interpolants.

(2.2). $\mathcal{P}$ is locally defined, i.e., evaluating the interpolant at a point within a tetrahedron domain requires only data defined on it. This ensures that local changes in the data will only have local effects.

Usually, the construction of a C^1 tetrahedral interpolant requires the positions and the first derivatives at vertices of the tetrahedron. But the above information is not sufficient to insure C^1 joins with adjacent interpolants. We restrict $\mathcal{P}$ to satisfy

(2.3). $\mathcal{P}$ interpolates some cross boundary derivatives in addition to the positions and the first derivatives at corners. In order to maintain the interpolation precision and shape fidelity, we insist that

(2.4). $\mathcal{P}$ has cubic precision, i.e., $\mathcal{P}$ reproduces any trivariate polynomial up to cubic degree.

Because a great deal of flexibility can be obtained by using rational functions, we also insist that

(2.5). $\mathcal{P}$ is a rational polynomial in form.

Finally, due to aesthetic reasons we insist that the form of the interpolant $\mathcal{P}$ is not affected by the ordering of the tetrahedron vertices, that is:

(2.6). $\mathcal{P}$ is of tetrahedral symmetric form.