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FINDING THE STRICTLY LOCAL AND ϵ-GLOBAL MINIMIZERS OF CONCAVE MINIMIZATION WITH LINEAR CONSTRAINTS*[1)]

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Abstract

This paper considers the concave minimization problem with linear constraints, proposes a technique which may avoid the unsuitable Karush-Kuhn-Tucker points, then combines this technique with Frank-Wolfe method and simplex method to form a pivoting method which can determine a strictly local minimizer of the problem in a finite number of iterations. Basing on strictly local minimizers, a new cutting plane method is proposed. Under some mild conditions, the new cutting plane method is proved to be finitely terminated at an ϵ-global minimizer of the problem.

1. Introduction

This paper considers the following nonlinear programming problem

$$(NLP) \qquad \min\{f(x) \mid x \in C\},$$

where $f(x)$ is a strictly concave function and $C \subset R^n$ is a convex polytope which will be specified later. It's well known that if (NLP) has a solution, then the minimum value can be attained at a vertex of the constraint. Generally speaking, this problem is NP-hard [1]. The ordinary descent methods usually generate a sequence of points which converges to a Karush-Kuhn-Tucker point of (NLP) under some conditions. Unfortunately, this Karush-Kuhn-Tucker point can not be guaranteed to be a local minimizer even if it satisfies the second order necessary conditions.

The purpose of this paper is to propose a technique for eliminating the unsuitable Karush–Kuhn–Tucker points. By combining this technique with Frank-Wolfe method and simplex method we form a descent method for (NLP). Under some mild conditions it is proved that, in a finite number of iterations, the method stops at a strictly local minimizer of (NLP). This kind of result was first obtained in [2] for a special class of problems they called concave knapsack problems. In their paper, they also gave out a tight complexity lower bound for their method. Although the global minimizer can not be guaranteed, the strictly local minimizer can provide good approximation to the global solution of (NLP) and they are very useful in the branch–and–bound

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algorithms for the global optimization. Basing on the strictly local minimizer, we will further present a new cutting plane method which can be viewed as a revised version of Tuy's cutting plane method [3].

The convergence of Tuy's cutting plane method is still an open problem except we add some extra conditions on the method itself [4], [5], [6]. The new cutting plane method uses an ϵ procedure and an alternative implicit vertex enumerating procedure and is therefore finitely convergent without any extra assumptions.

The paper will be organized as follows. In section 2 we will introduce some assumptions and notations; describe the finitely convergent algorithm for the strictly local minimizers and the corresponding convergence analysis. In section 3 we will present a new cutting plane method for the ϵ-global minimizer and its theoretical analysis. Section 4 will be the conclusion section.

2. Finding The Strictly Local Minimizer

This section considers the following concave minimization problem

$$(P) \qquad \min\{f(x) \mid x \in R\},$$

where $f(x)$ is a strictly concave function, $R = \{x|Ax = b, x \geq 0\}$, $A \in R^{m\times n}$, $b \in R^m$.

Throughout of this section, we will make and use the following assumptions and notations.

Assumption 1 *$f(x)$ is strictly concave and continuously differentiable.*

Assumption 2 *R is nonempty, bounded and rank$(A) = m$.*

Notations: $N = \{1, 2, \cdots, n\}$, $M = \{1, 2, \cdots, m\}$, $A = (a_{ij}|i \in M, j \in N)$. If $J \subseteq N$, $L \subseteq M$, then $A_L^J = (a_{ij}|i \in L, j \in J)$, when $J = N$ or $L = M$, we also simply set $A_L = A_L^N$ or $A^J = A_L^M$. For a given subset $I \subset N$ with $|I| = m$, $|*|$ designates the cardinality of $*$, if A^I is invertible, then set $T(I) = (A^I)^{-1}A$ and $t(I) = (A^I)^{-1}b$. If $t(I) \geq 0$, then I is called a basis. Let $\bar{I} = N \backslash I$, $T^{\bar{I}}(I) = (A^I)^{-1}A^{\bar{I}}$ and $T_r^{\bar{I}}$ is the rth row of $T^{\bar{I}}(I)$. For a given basis I and $x \in R$, let $x = (x_I, x_{\bar{I}})$, $\nabla f(x) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n}\right)$, $\nabla_{\bar{I}} f(x) = \left(\frac{\partial f}{\partial x_i}|i \in \bar{I}\right)$, $\nabla_I f(x) = \left(\frac{\partial f}{\partial x_i}|i \in I\right)$. It's clear that $x_I = t(I) - T^{\bar{I}}(I)x_{\bar{I}}$. If we define $\bar{f}(x_{\bar{I}}) = f(t(I) - T^{\bar{I}}(I)x_{\bar{I}}, x_{\bar{I}})$, then we have

$$\nabla \bar{f}(x_{\bar{I}}) = \nabla_{\bar{I}} f(x) - \nabla_I f(x) T^{\bar{I}}(I). \tag{1}$$

This formula just designates what is usually called the reduced gradient of $f(x)$. conv$(*)$ and vol$(*)$ will represent the convex hull of $*$ and the volume of $*$ respectively. $\emptyset$ denotes the empty set.

It can be seen that the above notations inherit that of the simplex method for linear programming except the cost vector now is $\nabla f(x)$. The following algorithm is designed for finding the strictly local minimizer of the problem (P).

Algorithm I

- Initilization

  Given a vertex x^0 of R, let I be its corresponding basis, set $k = 0$.

Step 1. Calculate $\nabla \bar{f}(x_{\bar{I}}^k)$ and $T^{\bar{I}}(I)$.