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LOCAL ARTIFICIAL BOUNDARY CONDITIONS FOR THE INCOMPRESSIBLE VISCOUS FLOW IN A SLIP CHANNEL*1)

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Abstract

In this paper we consider numerical simulation of incompressible viscous flow in an infinite slip channel. Local artificial boundary conditions at an artificial boundary are derived by the continuity of velocity and normal stress at the segment artificial boundary. Then the original problem is reduced to a boundary value problem on a bounded computational domain. Numerical example shows that our artificial boundary conditions are very effective.

1. Introduction

Many boundary value problems of partial differential equations involving unbounded domain occur in many areas of applications, e. g., fluid flow around obstacles, coupling of structures with foundation and so on. For getting the numerical solutions of the problems on unbounded domian, a natural approach is to cut off an unbounded part of the domain by introducing an artificial boundary and set up an appropriate artificial boundary condition on the artificial boundary. Then the original problem is approximated by a problem on bounded domain.

In the last ten years, boundary value problems in an unbounded domain have been studied by many authors. For instance, Goldstein [1], Feng [2], Han and Wu [3,4], Hagstrom and Keller [5,6], Halpern [7], Halpern and Schatzman [8], Nataf [9], Han, Lu and Bao [10], Han and Bao [11,12] and others have studied how to design artificial boudnary conditions for partial differential equations in an unbounded domain. Among their results, two kinds of artificial boundary conditions are designed. One is nonlocal artificial boundary condition, the other is local artificial boundary condition. In engineering, they like to use the second type.

In this paper we design local artificial boundary conditions for Navier-Stokes (N-S) equations in an infinite slip channel. Then the original problem is reduced to a boundary value problem in a bounded domain. Moreover numerical example shows that the artificial boundary conditions given in this paper are very effective.

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2. Navier-Stokes Equations and Oseen Equations

Let Ω_i be an obstruction in a channel defined by $\Re \times (0, L)$ and $\Omega = \Re \times (0, L) \setminus \bar{\Omega}_i$. Consider the following Navier-Stokes equations

$$(u \cdot \nabla)u + \nabla p = \nu \triangle u, \quad \text{in } \Omega, \tag{2.1}$$

$$\nabla \cdot u = 0, \quad \text{in } \Omega, \tag{2.2}$$

with boundary conditions

$$u_2|_{x_2=0,L} = 0, \quad \sigma_{12}|_{x_2=0,L} \equiv \nu\Big(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\Big)|_{x_2=0,L} = 0, \quad -\infty < x_1 < +\infty. \tag{2.3}$$

$$u|_{\partial\Omega_i} = 0, \tag{2.4}$$

$$u(x) \to u_\infty \equiv (\alpha, 0)^T, \quad \text{when } x_1 \to \pm\infty, \tag{2.5}$$

where $u = (u_1, u_2)^T$ is the velocity, p is the pressure, $\nu > 0$ is the kinematic viscosity, $x = (x_1, x_2)^T$ is coordinate, $\alpha > 0$ is a constant and σ_{12} is the tangential stress on the wall. Obviously condition (2.3) is equivalent to the following condition

$$\frac{\partial u_1}{\partial x_2}|_{x_2=0,L} = u_2|_{x_2=0,L} = 0, \quad -\infty < x_1 < +\infty. \tag{2.6}$$

Taking two constants $b < c$, such that $\Omega_i \subset (b, c) \times (0, L)$, then Ω is divided into three parts Ω_b, Ω_T and Ω_c by the artificial boundary $\Gamma_b = \{x \in \Re^2 \mid x_1 = b, 0 \le x_2 \le L\}$ and $\Gamma_c = \{x \in \Re^2 \mid x_1 = c, 0 \le x_2 \le L\}$ with

$$\Omega_b = \{x \in \Re^2 \mid -\infty < x_1 < b, \quad 0 < x_2 < L\},$$
$$\Omega_T = \{x \in \Re^2 \mid b < x_1 < c, \quad 0 < x_2 < L\} \setminus \overline{\Omega}_i,$$
$$\Omega_c = \{x \in \Re^2 \mid c < x_1 < +\infty, \quad 0 < x_2 < L\}.$$

When $|b|$ and c are sufficiently large, in the domain $\Omega_b \cup \Omega_c$ the velocity u is almost constant vector u_∞. So the N-S equations (2.1)–(2.2) can be linearized in domain Ω_c (and Ω_b), namely the solution (u, p) of problem (2.1)–(2.5) approximately satisfies the following problem

$$\alpha\frac{\partial u}{\partial x_1} + \nabla p = \nu \triangle u, \quad \text{in } \Omega_c, \tag{2.7}$$

$$\nabla \cdot u = 0, \quad \text{in } \Omega_c, \tag{2.8}$$

$$\frac{\partial u_1}{\partial x_2}|_{x_2=0,L} = u_2|_{x_2=0,L} = 0, \quad c \le x_1 < +\infty, \tag{2.9}$$

$$u(x) \to u_\infty = (\alpha, 0)^T, \quad \text{when } x_1 \to +\infty. \tag{2.10}$$

In [13], the author obtained general solution of the problem (2.7)–(2.10)

$$u_1(x) = \alpha + \sum_{m=1}^{\infty} \Big[a_m e^{-\frac{m\pi}{L}(x_1-c)} - \frac{m\pi}{L\lambda^-(m)} b_m e^{\lambda^-(m)(x_1-c)}\Big] \cos\frac{m\pi x_2}{L}, \tag{2.11}$$

$$u_2(x) = \sum_{m=1}^{\infty} \Big[a_m e^{-\frac{m\pi}{L}(x_1-c)} + b_m e^{\lambda^-(m)(x_1-c)}\Big] \sin\frac{m\pi x_2}{L}, \tag{2.12}$$