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MODIFIED SHAPE FUNCTIONS FOR SPECHT'S PLATE BENDING ELEMENT*1)

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Abstract

In this paper we discuss Specht's plate bending element, give the relationships between $\int_{F_\rho} w\,ds$ or $\int_{F_\rho} \frac{\partial w}{\partial n}\,ds$ and the nodal parameters (or freedoms of degrees), further light on the construction methods for that element and at last introduce a new plate bending element with good convergent properties (passing F-E-M-Test[11]).

1. Introduction

The solution of the C^1-continuity requirement of Kirchhoff bending with finite element models results in complicated higher elements[2],[4],[7]. Besides the large number of unknowns, difficulties may also arise from mixed second derivatives at the vertices taken as nodal variable[8]. To overcome such difficulties, a splitting spline element method is introduced[5],[9], but this always causes complicated computation. From the practical point of view lower-degree polynomial finite elements are more desirable. Unfortunately, the simple elements based on lower degree polynomials for the displacement field are non-conforming (not C^1 compatible). This may cause convergent problems and unreliable finite approximations. For non-conforming finite elements, there are some relaxed sufficient convergent conditions, such as the well-known patch test, the interpolation test, the generalized patch tests and the F-E-M-Test, instead of the strong C^1 continuity.

Consider the simple triangular plate bending element whose nodal variables (or freedoms of degree) are the deflection and two rotations at the vertices. Based on the quadratic displacement expansion proposed early by Zienkiewicz, this element is nonconforming because the normal slopes do not match continously along the interelement boundaries. As this element fails in the (generalized) patch test[10], Bergan in

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[1] proposed a modified displacement basis, but the modified version does not satisfy the patch test either. Later, by the aid of the interpolation test, B. Specht[13] constructed an appropriate polynomial displacement basis. This modified element passes the (generalized) patch test ensuring the convergence.

Specht's construction is based on the requirement of weak continuity, i.e., the displacement w and the normal slope $\frac{\partial w}{\partial n}$ (and tangent slope $\frac{\partial w}{\partial \tau}$) are continuous in the sense of intergral along the interelement boundaries. The intention of this article is to derive the relationships between $\int_{F_\rho} w\, ds$ as well as $\int_{F_\rho} \frac{\partial w}{\partial n} ds$ and the nodal variables, further to light on construction method for Specht's plate bending element and to introduce a new plate bending element with convergence by the aid of Shi's F-E-M-Test[5].

To facilitate our presentation, we must agree on certain notations. Given a triangle K with the vertices $P_i = (x_i, y_i)(i = 1, 2, 3)$ in counterclockwise order and the area Δ, we put

$$\xi_1 = x_2 - x_3, \qquad \xi_2 = x_3 - x_1, \qquad \xi_3 = x_1 - x_2,$$

$$\eta_1 = y_2 - y_3, \qquad \eta_2 = y_3 - y_1, \qquad \eta_3 = y_1 - y_2,$$

$$l_{12}^2 = \xi_3^2 + \eta_3^2, \quad l_{23}^2 = \xi_1^2 + \eta_1^2, \quad l_{31}^2 = \xi_2^2 + \eta_2^2,$$

$$r_1 = \frac{1}{\Delta}(\xi_2\xi_3 + \eta_2\eta_3), \ r_2 = \frac{1}{\Delta}(\xi_3\xi_1 + \eta_3\eta_1), \ r_3 = \frac{1}{\Delta}(\xi_1\xi_2 + \eta_1\eta_2),$$

$$t_1 = \frac{1}{\Delta}(\xi_1^2 + \eta_1^2), \ t_2 = \frac{1}{\Delta}(\xi_2^2 + \eta_2^2), \ t_3 = \frac{1}{\Delta}(\xi_3^2 + \eta_3^2).$$

Denote by F_i the edge of K opposite to the vertex P_i, and by τ_i and n_i the unit tangent and outward normal on $F_i (i = 1, 2, 3)$, respectively. Now we let λ_i denote the area coordinates relative to the vertices P_i, i.e.,

$$\begin{cases} x = x_1\lambda_1 + x_2\lambda_2 + x_3\lambda_3, \\ y = y_1\lambda_1 + y_2\lambda_2 + y_3\lambda_3, \\ 1 = \lambda_1 + \lambda_2 + \lambda_3 \end{cases}$$

such that the triangle K is transformed into the standard simplex $K^* = \{(\lambda_1, \lambda_2, \lambda_3) \mid \lambda_1 + \lambda_2 + \lambda_3 = 1, \lambda_i \geq 0\}$.

2. Analysis for Specht's Element

Specht's plate bending element was defined in [13] as follows.

Let K be a triangle with vertices at $P_i = (x_i, y_i), (i = 1, 2, 3)$ in counterclockwise order. Specht's element has three degrees of freedom per vertex, i.e., displacement at vertex and the two rotations expressed by the derivatives of the transverse displacement,