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A SPLITTING ITERATION METHOD FOR A SIMPLE CORANK-2 BIFURCATION PROBLEM*1)

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Abstract

A splitting iteration method is introduced to approximate a simple corank-2 bifurcation point of a nonlinear equation with small extended systems. This iteration method converges linearly with an adjustable speed and needs little extra computational work.

§1. Introduction

Let E be a Hilbert space, and $G : E \times I\!R$ a nonlinear C^3-mapping. We consider the nonlinear equation

$$G(u, \lambda) = 0 \tag{1.1}$$

and its corank-2 bifurcation problems. We assume that there is a point (u_0, λ_0) in $E \times I\!R$ satisfying

(H1) $$G_0 := G(u_0, \lambda_0) = 0$$

and

(H2) D_uG_0 is a Fredholm operator with index 0 and zero is one of its eigenvalues with algebraic multiplicity 2; furthermore,

$$\text{a)} \quad \dim(\text{Null}\,(D_uG_0)) = 2, \qquad \text{b)} \quad D_\lambda G_0 \in \text{Range}\,(D_uG_0). \tag{1.2}$$

The main aim of this paper is to introduce an efficient method for accurate approximation of the simple corank-2 bifurcation point (u_0, λ_0) of (1.1) and the null vectors of $D_uG_0, D_uG_0^*$ which are used in path following of (1.1) around (u_0, λ_0) (cf. [2, 7, 13, 15, 16]).

For highly singular problems of (1.1), Allgower and Böhmer[1], Beyn[2] and Mezel[10] have discussed some general principles on the extended systems; particularly, also see [18], [20] and [3] for simple corank-2 bifurcation problems. All these extended systems are at least three times larger than the original equation (1.1) and the equations in these

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systems are intrinsically dependent on each other. Consequently, the linearized equations in Newton-like iterations have to be solved directly or successively with the help of some intermediate unknowns, which leads to large computational efforts, especially for PDE problems.

On the other hand, by unfolding Rabier and Reddien[14] transformed the highly singular equation into a generalized turning point problem and then made up minimally extended systems with some implicitly defined scalar equations. But, the convergence of Newton's method and its implementation were not discussed.

We will set up various small extended systems for (1.1) and its linearized problem at the bifurcation point (u_0, λ_0), and introduce a splitting iteration method to approximate simultaneously the point (u_0, λ_0), the null vectors of $D_uG_0, D_uG_0^*$ in a coupled way. This iteration method converges with an adjustable speed and its computational cost at each iteration step remains at the same level as that for the regular solutions of (1.1).

In Section 2 we state the definition of a corank-2 bifurcation point type-I of (1.1) and show its base independent property. Section 3 discusses the splitting iteration method and its convergence. Finally, we present in Section 4 two simple numerical example showing the behaviour of the method.

§2. Corank-2 Type-I Bifurcation Points

We introduce in this section a corank-2 bifurcation point type-I of (1.1) and its base independent property. In the following, we assume that the conditions (H1) and (H2) are satisfied and the mapping G is C^3-continuous. We see easily from statement (1.2b) that

$$\dim(N(DG_0)) = 3.$$

On the other hand, if

$$D_\lambda G_0 \notin R(D_uG_0),$$

equation (1.1) can be transformed into a simple bifurcation problem under symmetries and other parametrizations[19].

Under the conditions (H1)–(H2), the Fredholm operator theory shows that there are elements $\phi_i, \phi_i^* \in E, i = 1, 2$, such that

$$\begin{cases} V_1 := N(D_uG_0) = \operatorname{Span}[\phi_1, \phi_2], \langle\phi_1, \phi_j\rangle = \delta_{ij}, \\ \tilde{V}_1 := N(D_uG_0^*) = \operatorname{Span}[\phi_1^*, \phi_2^*], \langle\phi_i^*, \phi_j\rangle = \delta_{ij}, i, j = 1, 2, \end{cases} \tag{2.1}$$

and

$$\begin{cases} V_2 := R(D_uG_0) = \{u \in E, \langle\phi_i^*, u\rangle = 0, \quad i = 1, 2\} \\ \tilde{V}_2 := R(D_uG_0) = \{u \in E, \langle\phi_i, u\rangle = 0, \quad i = 1, 2\}; \end{cases} \tag{2.2}$$

furthermore,

$$E = V_1 \oplus V_2 = \tilde{V}_1 \oplus \tilde{V}_2, \tag{2.3}$$