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SEMI–COARSENING IN MULTIGRID SOLUTION OF STEADY INCOMPRESSIBLE NAVIER–STOKES EQUATIONS*

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Abstract

We present a semi–coarsening procedure, i. e., coarsening in one space direction, to improve the convergence rate of the multigrid solver presented in [5] for solving the 2D steady Navier–Stokes equations in primitive variables when the aspect ratio of grid cells is not equal to 1, i. e., when $h_x/h_y \gg 1$ or $\ll 1$, where h_x is the grid step in x direction and h_y the grid step in y direction, x and y represent the Cartesian coordinates.

Introduction

In numerical simulation of fluid flows we encounter frequently situations in which physical quantities (pressure, velocity, etc.) change at different scales in different space directions. When there is a main flow direction, where the change of physical quantities is much smaller along the flow direction than in the direction orthogonal to the main flow, different grid steps in different directions are often used. For dealing with these problems, it is essential to have a solver of the discrete system whose convergence rate is not very sensitive to the ratio of grid steps.

In [5], we have presented a multigrid solver for solving the 2D steady Navier–Stokes equations in primitive variables on rectangular regions. It is based on second–order up-wind differencing for the discretization of the convection terms and the SCGS relaxation procedure (this procedure was originally proposed by S.P. Vanka[3] as smoothing operator for his multigrid solver based on hybrid differencing) and has been observed to have good convergence rate for Reynolds numbers up to 10000*.

If we denote by h_x the grid step in x direction and h_y the grid step in y direction, the convergence rate of the above M.G. solver depends on the ratio $\rho \stackrel{\text{def}}{=} h_x/h_y$. The best convergence rate is obtained when $\rho = 1$ while the convergence rate is significantly slowed down when $\rho \ll 1$ or $\rho \gg 1$, as can be seen through Figure 1, which shows the total residual of the approximate solution with regard to the number of multigrid iterations. In this example, the computational region is the rectangle $(0, A) \times (0, B)$ and the grid is the 32×32 uniform grid, so $\rho = A/B$. The four curves are obtained with $A = 1$ and $B = 1$, 4, 8, 16, respectively and the following test solution :

$$\begin{aligned} u(x,y) &= A\sin(\frac{x}{A})\cos(\frac{y}{B}), \\ v(x,y) &= -B\cos(\frac{x}{A})\sin(\frac{y}{B}), \\ p(x,y) &= \frac{x}{A}\frac{y}{B}. \end{aligned}$$

The V-cycle is used with 2 pre–relaxations, 1 post–relaxation in the multigrid solver and the relaxation parameter $\beta = 0.8$. The multigrid procedure converges when ρ $geq1/4$ but

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the convergence is very slow when $\rho = 1/4$ and it even diverges when $\rho \leq 1/8$ (we can get convergence when $\rho \geq 1/8$ by using smaller relaxation parameter β but the convergence is always very slow).

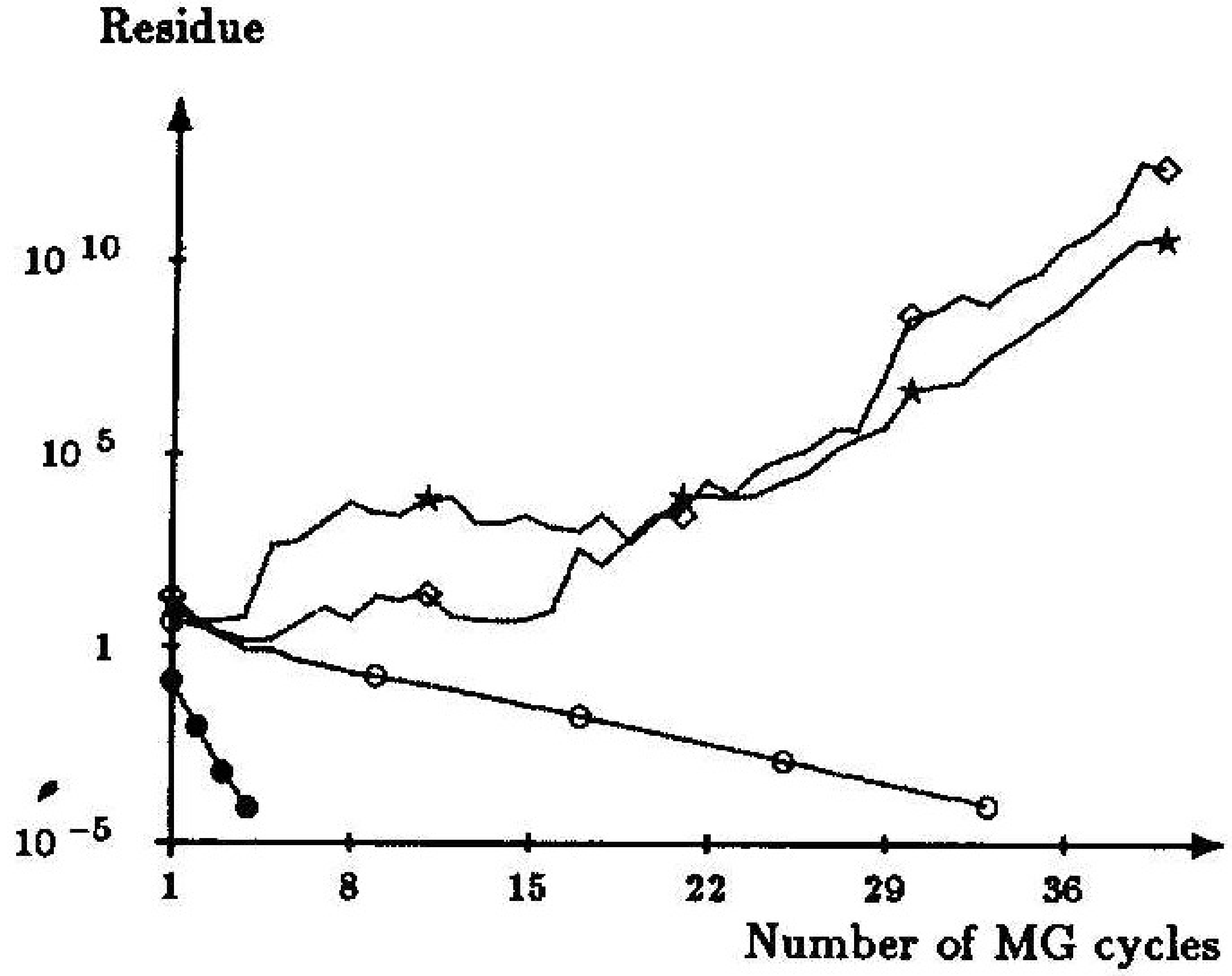

Figure 1: Convergence rate of the MG solver for the test problem ($R = 100$) with different values of ρ :
$\bullet : \rho = 1$ $\circ : \rho = 1/4$ $\star : \rho = 1/8$ $\diamond : \rho = 1/16$

The convergence rate of multigrid solvers depends essentially on the smoothing properties of the relaxation procedure (also called smoothing operator). The SCGS relaxation has better smoothing properties when grid cells are nearly square. An easy way to improve the convergence rate when $\rho \gg 1$ or $\rho \ll 1$ is to use coarse grids obtained by increasing only one grid step in one space direction, instead of increasing the grid steps in all directions, in the multigrid procedure (this approach was also proposed by Hackbusch[2] for solving anisotropic problems). The object of this study is to investigate the efficiency of the semi-coarsening in multigrid solution of steady Navier–Stokes equations.

In the present paper, we will first recall briefly the multigrid solver presented in [5]. Then we give details of the implementation of the coarsening procedure and corresponding numerical results.

Remark. Vanka has also done some numerical experiments with the second–order upwind differencing. Contrast to our conclusions given in [1] and [5], he has observed very slow convergence of his multigrid solver combined with the second–order upwind differencing and no improvement in the precision of approximation with regard to the hybrid scheme, when Reynolds number is greater than or equal to 600 (see [4]). There are several differences between his scheme and the ours which may be the cause of slow convergence and poor precision of his scheme :

1. He wrote the convection terms discretised by second–order upwinding finite differences as the corresponding first–order upwinding discretization multiplied by a constant plus a