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GENERALIZED INVERSION AND ITS APPLICATION IN INVERSE SCATTERING PROBLEMS *

Zhang Xian-kang

(*Geophysical Prospecting Brigade, State Seismological Bureau, Zhengzhou, China*)

Abstract

In this paper, the generalized inversion theory and its application in inverse scattering problems are discussed. An iterative solution of joint inversion of parameters describing the earth structures and sources is given and a numerical example is also shown.

Introduction

The inversion problem is a quite active field of research in geophysics. Many geophysical problems are regarded as reconstruction of the spatial distribution of some physical parameters from the images of the model space in the data space [1], [2], [3]. The reconstruction of the earth structures from the reflections observed at the surface is just one of this kind of problems. Based on the generalized inversion and by applying Born approximation, an iterative solution of linear inversion of reflections has been obtained by A. Tarantola et al. [3], [4], [5], [7]. Their result of the first iteration just corresponds to the classical migration. It is also possible to regard the source function as an unknown parameter. An iterative solution of simultaneous inversion of parameters describing the earth structures and sources is obtained in this paper.

§1. Theory

The inversion problems in geophysics are generally ill-posed. In particular, when we attempt to discretise model parameters not at the beginning of formula establishment, but at the last step of calculations, the general inversion theory based on the matrix algorithm would no longer be sufficient. The generalized inversion method provides the basis for solving this kind of inversion problems [3], [6].

1. Data space and model space

The functional space which consists of all acceptable models is called the model space and represented by M; the vector space which consists of all observable data is called the data space and denoted by D. The real line is represented by R.

Let the weight functions of M and D be $W_m(r,r')$ and $W_d(r,r')$ respectively.

The norm of the model space, $\|\ \|_M : M \to R$, is determined by

$$\|\underline{m}\|_M = \left\{ \int dr \int dr' m(r) W_m(r,r') m(r') \right\}^{1/2}, \quad \underline{m} \in M. \tag{1.1}$$

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The norm of the data space, $\| \ \|_D : D \to R$, is determined by

$$\|\underline{d}\|_D = \left\{ \int dr \int dr' d(r) W_d(r, r') d(r') \right\}^{1/2}, \quad \underline{d} \in D. \tag{1.2}$$

Define the scalar products of the model space and the data space as $\langle \ , \ \rangle_M : M \times M \to R$, and

$$\langle \underline{m}_1, \underline{m}_2 \rangle_M = \int dr \int dr' m_1(r) W_m(r, r') m_2(r'), \quad \underline{m}_1, \underline{m}_2 \in M; \tag{1.3}$$

$\langle \ , \ \rangle_D : D \times D \to R$, and

$$\langle \underline{d}_1, \underline{d}_2 \rangle = \int dr \int dr' d_1(r) W_d(r, r') d_2(r'), \quad \underline{d}_1, \underline{d}_2 \in D. \tag{1.4}$$

We have

$$\begin{aligned} \|\underline{d}\|_D^2 &= \langle \underline{d}, \underline{d} \rangle_D, \quad \underline{d} \in D, \\ \|\underline{m}\|_M^2 &= \langle \underline{m}, \underline{m} \rangle_M, \quad \underline{m} \in M. \end{aligned} \tag{1.5}$$

Let the dual spaces of model and data spaces be denoted by $\hat{M}$ and $\hat{D}$ respectively. From the definition of scalar products, the elements in the dual spaces can be correlated with the elements in the original spaces.

$$\begin{aligned} \hat{m}_1(r) &= \int dr' W_m(r, r') m_1(r), \quad \underline{m}_1 \in M, \quad \hat{\underline{m}}_1 \in \hat{M}, \\ \hat{d}_1(r) &= \int dr' W_d(r, r') d_1(r), \quad \underline{d}_1 \in D, \quad \hat{\underline{d}}_1 \in \hat{D}. \end{aligned} \tag{1.6}$$

Introduce the definition of transposed operator [3], [6]: the transposed operator $\underline{G}^T$ of the linear operator $\underline{G} : M \to D$, maps $\hat{D}$ into $\hat{M}$ and meets the following relation:

$$\langle \underline{Gm}, \underline{d} \rangle_D = \langle \hat{\underline{m}}, \underline{G}^T \hat{\underline{d}} \rangle_{\hat{M}}, \quad \underline{m} \in M, \underline{d} \in \underline{D}, \quad \hat{\underline{m}} \in \hat{M}, \quad \hat{\underline{d}} \in \hat{D}. \tag{1.7}$$

The introduction of the transposed operator will play an important role in the generalized inversion theory. It allows us to compare the inversion problem of functionals with that of discrete parameters in many cases.

If for $\underline{G} : M \to D$ there is an operator $\underline{G}^* : D \to M$ and it meets $\langle \underline{d}, \underline{Gm} \rangle_D = \langle \underline{G}^* \underline{d}, \underline{m} \rangle_M, \underline{d} \in D, \underline{m} \in M, \underline{G}^*$ is called the adjoint operator of $\underline{G}$.

If a symmetric linear and positive definite operator $\underline{C}_m : \hat{M} \to M$ exists, it is called the covariance operator of the model space; in the same way, $\underline{C}_d : \hat{D} \to D$ is called the covariance operator of the data space. Their inverse operators always exist. If the weight functions of M and $D, W_m(r, r')$ and $W_d(r, r')$, are just the kernels of $\underline{C}_m^{-1}$ and $\underline{C}_d^{-1}$ respectively, we call the scalar products defined in this way the natural scalar products [6]. Throughout this paper, we shall define the scalar products in this way. Thus, we have

$$\hat{\underline{m}} = \underline{C}_m^{-1} \underline{m}, \quad \underline{m} \in M, \quad \hat{\underline{m}} \in \hat{M}, \quad \hat{\underline{d}} = \underline{C}_d^{-1} \underline{d}, \quad \underline{d} \in D, \quad \hat{\underline{d}} \in \hat{D}. \tag{1.8}$$

Consequently, $\hat{M}$ and $\hat{D}$ can be regarded as the image space of $\underline{C}_m^{-1}$ and $\underline{C}_d^{-1}$.