THE SECOND-ORDER FLUID IN CELL (FLIC) METHOD FOR THE ONE-DIMENSIONAL UNSTEADY COMPRESSIBLE FLOW PROBLEMS*

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Abstract

In this paper we suggest a second-order fluid in cell (FLIC) method for the one-dimensional unsteady compressible flow problems. The Numerical result obtained by the present method is compared with the one obtained with the original FLIC method and the exact solution for a shock tube problem.

Introduction

The fluid in cell (FLIC) method[1,2] is one of the most useful difference methods in the computational fluid dynamics. However, as it has only first-order accuracy, it cannot give a satisfactory numerical result in some cases. This paper suggests a second-order fluid in cell (FLIC) method for the one-dimensional unsteady compressible flow problems. The result obtained by the present method is compared with the one obtained with the original FLIC method and the exact solution for the shock tube problem. The comparison demonstrates that the second-order FLIC method is satisfactory.

Second-order Fluid in Cell Method

The equations of one-dimensional unsteady compressible fluid flow may be written in the following forms:

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+\rho\frac{\partial u}{\partial x}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0, \tag{2}$$

$$\frac{\partial e}{\partial t}+u\frac{\partial e}{\partial x}+\frac{p}{\rho}\frac{\partial u}{\partial x}=0, \tag{3}$$

where ρ is the density, u is the velocity, p is the pressure and e is the internal energy per unit mass. Assume the gas is polytropic; in that case equation of state is

$$p=(r-1)\rho e, \tag{4}$$

where r is a constant greater than one.

Let Δx and Δt, respectively, denote the spatial increment and the time increment.

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The second-order FLIC method calculates the quantities at time $(n+1)\Delta t$ in terms of those at time $n\Delta t$ where n is a number of time step. Within one time step, the new quantities are computed in two phases: First, intermediate values are computed for the velocities and specific internal energy, taking into account the effects of acceleration caused by pressure gradients. Second, transport effects are computed.

Phase 1. By the following two-step method, intermediate values $\tilde{u}_i$ and $\tilde{e}_i$ are obtained:

Step 1:

$$\tilde{\tilde{u}}_i = u_i^n - \frac{\Delta t}{2\Delta x}\frac{1}{\rho_i^n}(p_{i+1/2}^n - p_{i-1/2}^n), \tag{5}$$

$$\tilde{\tilde{e}}_i = e_i^n - \frac{\Delta t}{2\Delta x} p_i^n \frac{1}{\rho_i^n}(u_{i+1/2}^n - u_{i-1/2}^n), \tag{6}$$

$$\tilde{\tilde{p}}_i = (r-1)\rho_i^n \tilde{\tilde{e}}_i. \tag{7}$$

Step 2:

$$\tilde{u}_i = u_i^n - \frac{\Delta t}{\Delta x}\frac{1}{\rho_i^n}(\tilde{\tilde{p}}_{i+1/2} - \tilde{\tilde{p}}_{i-1/2}), \tag{8}$$

$$\tilde{e}_i = e_i^n - \frac{\Delta t}{\Delta x}\frac{\tilde{\tilde{p}}_i}{\rho_i^n}(\tilde{\tilde{u}}_{i+1/2} - \tilde{\tilde{u}}_{i-1/2}). \tag{9}$$

Here
$$p_{i\pm1/2}^n = \frac{1}{2}(p_i^n + p_{i\pm1}^n), \quad u_{i\pm1/2}^n = \frac{1}{2}(u_i^n + u_{i\pm1}^n),$$
$$\tilde{\tilde{p}}_{i\pm1/2} = \frac{1}{2}(\tilde{\tilde{p}}_i \pm \tilde{\tilde{p}}_{i\pm1/2}), \quad \tilde{\tilde{u}}_{i\pm1/2} = \frac{1}{2}(\tilde{\tilde{u}}_i \pm \tilde{\tilde{u}}_{i\pm1/2}).$$

Phase 2. Transport effects are now computed. We regard the distributions of intermediate values of ρ_i, $\tilde{u}_i$, $\tilde{e}_i$ in each mesh $(x_{i-1/2}, x_{i+1/2})$ as linear functions, i.e.

$$\begin{cases} \rho = \rho_i + (x - x_i)\dfrac{\Delta_i \rho}{\Delta x}, \\ \tilde{u} = \tilde{u}_i + (x - x_i)\dfrac{\Delta_i \tilde{u}}{\Delta x}, \quad x_{i-1/2} < x < x_{i+1/2}, \\ \tilde{e} = \tilde{e}_i + (x - x_i)\dfrac{\Delta_i \tilde{e}}{\Delta x}, \end{cases} \tag{10}$$

where

$$\begin{cases} x_{i-1/2} = x_i - \dfrac{\Delta x}{2}, \\ x_{i+1/2} = x_i + \dfrac{\Delta x}{2}, \\ \Delta_i \rho = \dfrac{1}{2}(\rho_{i+1} - \rho_{i-1}), \\ \Delta_i \tilde{u} = \dfrac{1}{2}(\tilde{u}_{i+1} - \tilde{u}_{i-1}), \\ \Delta_i \tilde{e} = \dfrac{1}{2}(\tilde{e}_{i+1} - \tilde{e}_{i-1}). \end{cases} \tag{11}$$

In order to preserve the monotonicity of the numerical solution, the following van Leer monotonicity algorithm[3] is used: