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A DIFFERENCE SCHEME FOR SOLVING AN INITIAL VALUE PROBLEM FROM SEMICONDUCTOR DEVICE THEORY*1)

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I. Introduction

We consider the following problem from the semiconductor device theory:

$$\begin{aligned}
&\frac{\partial U}{\partial t}=\nu\sum_{i=1}^{n}\frac{\partial^2 U}{\partial x_i^2}-\nabla\cdot(U\nabla\psi)-R(U, V), \quad (x, t)\in\Omega\times(0, T],\\
&\frac{\partial V}{\partial t}=\nu\sum_{i=1}^{n}\frac{\partial^2 V}{\partial x_i^2}+\nabla\cdot(V\nabla\psi)-R(U, V), \quad (x, t)\in\Omega\times(0, T],\\
&p\sum_{i=1}^{n}\frac{\partial^2\psi}{\partial x_i^2}=U-V-N, \quad (x, t)\in\Omega\times(0, T],\\
&\frac{\partial U}{\partial n}=\frac{\partial V}{\partial n}=\frac{\partial\psi}{\partial n}=0, \quad (x, t)\in\Gamma\times[0, T],\\
&U(x, 0)=U_0(x), \quad x\in\Omega,\\
&V(x, 0)=V_0(x), \quad x\in\Omega,
\end{aligned} \tag{1.1}$$

where $x=(x_1, x_2, \cdots, x_n)$, $\Omega=\{X\,|\,0<x_j<L,\ 1\leqslant j\leqslant n\}$, Γ is the boundary of Ω, T is a specified positive constant, ν, p, q are positive constants, N is a specified Hölder continuous function of x, t,

$$R(U, V)=\frac{UV-1}{q(U+V+2)}, \tag{1.2}$$

$U_0(x), V_0(x)$ are twice continuously differentiable in x and strictly positive in $\Omega+\Gamma$, and

$$\int_\Omega (U_0(x)-V_0(x)-N(x))dx=0. \tag{1.3}$$

By a solution, we mean a set of three functions U, V, ψ of (x, t) in $\Omega\times[0, T]$, twice continuously differentiable in x and continuously differentiable in t satisfying (1.1)—(1.3), with U and V positive. For uniqueness we require also

$$\int_\Omega \psi(x, t)dx=0, \quad \forall t\in[0, T].$$

Mock[1] proved that, under the above conditions, (1.1) has a unique solution, and gave a difference scheme for solving (1.1), but without the proof of convergence.

In this paper we give a scheme for solving (1,1) with a strict proof of its convergence.

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1) This work is suggested by Professor R. Glowinski.

II. Notations and Lemmas

Let h be the mesh size in variable x_j, $j=1, 2, \cdots, n$. Q denotes a mesh point, and e_j is a unit vector, i. e.

$$e_j=(\underbrace{0, 0, \cdots, 0}_{j-1}, 1, \underbrace{0, \cdots, 0}_{n-j})^T.$$

Ω_h denotes the set of internal mesh points. Γ_h is the boundary of Ω_h,

$$\Gamma_{jM}=\{Q|Q\in\Gamma_h,\ Q-he_j\in\Omega_h\},$$
$$\Gamma_{jm}=\{Q|Q\in\Gamma_h,\ Q+he_j\in\Omega_h\},$$
$$\Gamma_j=\Gamma_{jM}+\Gamma_{jm}.$$

τ denotes the mesh size of variable t, $\lambda=\tau h^{-2}$.

Let η be the discrete function. $\eta(Q, k)$ denotes the value of η at point Q and time $t=k\tau$.

$$\eta(k)=\{\eta(Q, k)/Q\in\Omega_h+\Gamma_h\}.$$

For simplicity, we denote $\eta(Q, k)$ by $\eta(Q)$ or $\eta(k)$. We define

$$\eta_{x_j}(Q, k)=\frac{1}{h}[\eta(Q+he_j, k)-\eta(Q, k)],$$

$$\eta_{\bar{x}_j}(Q, k)=\frac{1}{h}[\eta(Q, k)-\eta(Q-he_j, k)],$$

$$\eta_n(Q, k)=\begin{cases}\eta_{\bar{x}_j}(Q, k), & \text{if } Q\in\Gamma_{jM},\\ -\eta_{x_j}(Q, k), & \text{if } Q\in\Gamma_{jm},\end{cases}$$

$$\Delta_{x_j}\eta(Q, k)=\eta_{\bar{x}_jx_j}(Q, k),\quad \Delta\eta(Q, k)=\sum_{j=1}^{n}\Delta x_j\eta(Q, k),$$

$$\eta_t(Q, k)=\frac{1}{\tau}[\eta(Q, k+1)-\eta(Q, k)],$$

and define the following scalar product and norms

$$(\eta, \xi)=\sum_{Q\in\Omega_h}h^n\eta(Q)\xi(Q),$$

$$\|\eta\|^2=(\eta, \eta),$$

$$\|\eta\|_1^2=\frac{1}{2}\sum_{j=1}^{n}(\|\eta_{x_j}\|^2+\|\eta_{\bar{x}_j}\|^2),$$

$$|\eta|_{\Gamma_h}=\max_{Q\in\Gamma_h}|\eta(Q)|,$$

$$\|\eta\|_{\Gamma_h}^2=\sum_{Q\in\Gamma_h}h^{n-1}\eta^2(Q).$$

We will use the following lemmas.

Lemma 1. $2(\eta, \eta_t)=(\|\eta\|^2)_t-\tau\|\eta_t\|^2.$

Lemma 2.

$$(\eta, \xi_{x_j})+(\xi, \eta_{x_j})=h^{n-1}\sum_{Q\in\Gamma_{jM}}\eta(Q-he_j)\xi(Q)-h^{n-1}\sum_{Q\in\Gamma_{jm}}\eta(Q)\xi(Q+he_j), \tag{2.1}$$

$$(\eta, \xi_{\bar{x}_j})+(\xi, \eta_{x_j})=h^{n-1}\sum_{Q\in\Gamma_{jM}}\eta(Q)\xi(Q-he_j)-h^{n-1}\sum_{Q\in\Gamma_{jm}}\eta(Q+he_j)\xi(Q). \tag{2.2}$$

The proofs come from Abel's formula directly.

Lemma 3.