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Application of Improved (G'/G)–Expansion Method to Traveling Wave Solutions of Two Nonlinear Evolution Equations

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Abstract. In this work, the improved (G'/G)-expansion method is proposed for constructing more general exact solutions of nonlinear evolution equation with the aid of symbolic computation. In order to illustrate the validity of the method we choose the RLW equation and SRLW equation. As a result, many new and more general exact solutions have been obtained for the equations. We will compare our solutions with those gained by the other authors.

AMS subject classifications: 35C07, 35C08, 35A25

Key words: RLW equation, SRLW equation, improved (G'/G)-expansion method, traveling wave solution.

1 Introduction

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical physical sciences such as physics, biology, chemistry. The investigation of traveling wave solutions to nonlinear evolution equations play an important role in the study of nonlinear physical phenomena.

Many effective methods [1–10] have been presented such as, inverse scattering transform method [1], Hirota's method [2], variational iteration method [3], the homogeneous balance method [4], Backlund and Darboux transformation method [5], the sine-cosine function method [6], the Jacobi elliptic function method [7], auxiliary equation method [8–10] and others.

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Recently, the (G'/G)-expansion method, firstly introduced by wang et al. [11] has become widely used to search for various exact solutions of NLEEs [12–16]. The value of the (G'/G)-expansion method is that one treats nonlinear problems by essentially linear methods. Very lately to enhance the (G'/G)-expansion method and expand the range of its applicability, further research has been carried out by several authors. Zhang et al. [14] improved the method to deal with the (2+1)-dimensional Broer-Kaup equation with variable coefficients. Shehata [15] modified the method to derive traveling wave solutions for nonlinear Schrodinger equation and the cubic-quintic Ginzburg Landau equation. Zhang [17] explored a new application of this method to some special nonlinear evolution equations, the balance numbers of which are not positive integers.

(G'/G)-expansion method and the transformed rational function method used by W. X. Ma [18, 19] have a common idea. That is, we firstly put the given NLEE into the corresponding ordinary differential equation (ODE), then the ODE can be transformed into a systems of of algebraic polynomials with the determining constants. By the solutions of the ODE, we can obtain the exact traveling solutions and rational solution of the NLEE. However, to get the N-soliton and N-wave solution of the PDE, we may consider the linear superposition principle [20] and multiple exp-function method [21], these methods can be applied to the Hirota bilinear equations and others. Ma [20, 21] have obtained many N-wave solutions of the (3+1)-dimensional potential-Yu-Toda-Sasa-Fukuyama equation, the (3+1)-dimensional KP equations et al. There is on application of (G'/G)-expansion method in this area so far.

In this paper, we will propose the improved (G'/G)-expansion method to construct more general exact solutions of nonlinear evolution equations (NLEES). For illustration, we restrict our attention to the study of Regularized long wave (RLW) equation and Symmetric RLW equation and successfully construct many new and more general exact solutions.

2 Description of the improved (G'/G)-expansion method

Suppose that we have a NLEE for $u(x,t)$ in the form

$$P(u, u_x, u_t, u_{xt}, u_{xx}, \cdots) = 0, \tag{2.1}$$

where P is a polynomial in its arguments, which includes nonlinear terms and the highest order derivatives. The transformation $u(x,t) = u(\xi)$, $\xi = x - \omega t$ reduces Eq. (2.1) to the ordinary differential equation (ODE)

$$H(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \cdots) = 0. \tag{2.2}$$

By virtue of the extended tanh-function method, we assume that the solution of Eq. (2.2) is of the form

$$u(\xi) = \sum_{i=0}^{l} a_i F^i(\xi), \tag{2.3}$$

in which a_i $(i = 0, 1, 2, \cdots, l)$, $\xi = x - \omega t$, ω is wave velocity, are all real constants to be determined, the balancing number l is a positive integer which can be determined by balancing the highest derivative terms with the highest power nonlinear terms in Eq. (2.2) and $F(\xi)$ is

$$F(\xi) = \frac{G'(\xi)}{G(\xi)}, \tag{2.4}$$

where $G(\xi)$ expresses the solution of the following auxiliary ordinary differential equation

$$GG'' = AG^2 + BGG' + C(G')^2, \tag{2.5}$$

where the prime denotes derivative with respect to ξ. A, B, C are real parameters.

Using the general solutions of Eq. (2.5), with the help of Maple we have the following four solutions of Eq. (2.4):

Case 1. when $B \neq 0$ and $\triangle = B^2 + 4A - 4AC \geq 0$, then

$$F(\xi) = \frac{B}{2(1-C)} + \frac{B\sqrt{\triangle}}{2(1-C)} \frac{c_1 e^{\frac{\sqrt{\triangle}}{2}\xi} + c_2 e^{-\frac{\sqrt{\triangle}}{2}\xi}}{c_1 e^{\frac{\sqrt{\triangle}}{2}\xi} - c_2 e^{-\frac{\sqrt{\triangle}}{2}\xi}}. \tag{2.6}$$

Case 2. when $B \neq 0$ and $\triangle = B^2 + 4A - 4AC < 0$, then

$$F(\xi) = \frac{B}{2(1-C)} + \frac{B\sqrt{-\triangle}}{2(1-C)} \frac{ic_1 \cos\frac{\sqrt{-\triangle}}{2}\xi - c_2 \sin\frac{\sqrt{-\triangle}}{2}\xi}{ic_1 \sin\frac{\sqrt{-\triangle}}{2}\xi + c_2 \cos\frac{\sqrt{-\triangle}}{2}\xi}. \tag{2.7}$$

Case 3. when $B = 0$ and $\triangle = A(C-1) \geq 0$, then

$$F(\xi) = \frac{\sqrt{\triangle}}{(1-C)} \frac{c_1 \cos(\sqrt{\triangle}\xi) + c_2 \sin(\sqrt{\triangle}\xi)}{c_1 \sin(\sqrt{\triangle}\xi) - c_2 \cos(\sqrt{\triangle}\xi)}. \tag{2.8}$$

Case 4. when $B = 0$ and $\triangle = A(C-1) < 0$, then

$$F(\xi) = \frac{\sqrt{-\triangle}}{(1-C)} \frac{ic_1 \cosh(\sqrt{-\triangle}\xi) - c_2 \sinh(\sqrt{-\triangle}\xi)}{ic_1 \sinh(\sqrt{-\triangle}\xi) - c_2 \cosh(\sqrt{-\triangle}\xi)}. \tag{2.9}$$

where $\xi = x - wt$. w is wave velocity, A, B, C and c_1, c_2 are real parameters.

3 Applications

In this section, we will demonstrate the improved (G'/G)-expansion method on two well-known nonlinear evolution equations, namely the generalized regularized long wave (RLW) equation an Symmetric regularized long wave (SRLW) equation.

3.1 Generalized regularized long wave (RLW) equation

The generalized regularized long wave (RLW) equation read

$$u_t + u_x + a(u^2)_x - bu_{xxt} = 0, \tag{3.1}$$

where a and b are positive constants. Eq. (3.1) was first put forward as a model for small amplitude long waves on the surface of water in a channel by Peregrine [22] and later by Benjamin [23]. In physical situations such as unidirectional waves propagational in water channel, long crested waves in near shore zones and many other, the generalized RLW equation serves an alternative model to the KDV equations.

Suppose $u = u(\xi) = u(x - wt)$, then Eq. (3.1) is carried to

$$-wu' + u' + 2auu' + bwu''' = 0, \tag{3.2}$$

the prime denotes the derivative with respect to ξ. Integrating Eq. (3.2) once with respect to ξ and setting the integration constant as zero yields

$$-wu + u + au^2 + bwu'' = 0. \tag{3.3}$$

Balancing the highest order nonlinear term u^2 and the highest order partial derivative u'', we get $l + 2 = 2l$, hence $l = 2$. So we can suppose that Eq. (3.3) has the following ansatz:

$$u(\xi) = a_0 + a_1 F(\xi) + a_2 F^2(\xi), \tag{3.4}$$

where a_0, a_1, a_2 are constants and need to be determined, $F(\xi)$ express the solution of Eq. (2.4). Substituting Eqs. (3.4) and (2.5), along with (2.4) into Eq. (3.3) and using Maple yields a system of equations of $(G'/G)^i$, setting the coefficients of $(G'/G)^i$ $(i = 0, 1, 2, \cdots, 4)$ in the obtained system of equations to zero, we can deduce the following set of algebraic polynomials with the respect unknowns a_0, a_1, a_2 namely:

$$\begin{aligned}
&6bwa_2 + aa_2^2 + 6bwa_2C^2 - 12bwCa_2 = 0,\\
&2aa_1a_2 + 10bwBCa_2 + 2bwC^2a_1 - 4bwCa_1 + 2bwa_1 - 10bwBa_2 = 0,\\
&3bwBCa_1 - wa_2 + aa_1^2 + 4bwa_2B^2 + 2aa_0a_2 + 8bwACa_2 + a_2 - 3bwBa_1 - 8bwAa_2 = 0,\\
&a_1 - wa_1 + 6bwABa_2 + 2aa_0a_1 - 2bwAa_1 + bwB^2a_1 + 2bwa_1AC = 0,\\
&aa_0^2 + a_0 + 2bwA^2a_2 - wa_0 + bwa_1AB = 0.
\end{aligned}$$

Solving the set of the above algebraic equations by use of Maple. We get the following results:

Case 1.

$$\left\{\begin{aligned}
&w = -\frac{1}{4bAC - 1 - 4bA - bB^2}, \quad a_0 = \frac{b(2AC + B^2 - 2A)}{a(4bAC - 1 - 4bA - bB^2)},\\
&a_1 = \frac{6Bb(C-1)}{a(4bAC - 1 - 4bA - bB^2)}, \quad a_2 = \frac{6b(C^2 - 2C + 1)}{a(4bAC - 1 - 4bA - bB^2)}.
\end{aligned}\right\}.$$

Case 2.

$$\left\{\begin{array}{ll} w=\dfrac{1}{4bAC+1-4bA-bB^2}, & a_0=-\dfrac{6bA(C-1)}{a(4bAC+1-4bA-bB^2)}, \\ a_1=-\dfrac{6Bb(C-1)}{a(4bAC+1-4bA-bB^2)}, & a_2=-\dfrac{6b(C^2-2C+1)}{a(4bAC+1-4bA-bB^2)}. \end{array}\right\}.$$

Substituting Case 1 and Case 2 in (3.4) and according to (2.6), (2.7), (2.8) and (2.9), we obtain the following exponential function solutions, hyperbolic function solutions and triangular function solutions of Eq. (3.1). These solutions are:

(1) When we choose $B \neq 0$ and $\triangle_1 = B^2+4A-4AC \geq 0$, then the exponential function solutions can be found as

$$u(x,t)=\frac{\varepsilon bw\triangle_1}{2a}-\frac{3B^2bw\triangle_1}{2a}\Big[\frac{c_1e^{\frac{\sqrt{\triangle_1}}{2}(x-\omega t)}+c_2e^{-\frac{\sqrt{\triangle_1}}{2}(x-\omega t)}}{c_1e^{\frac{\sqrt{\triangle_1}}{2}(x-\omega t)}-c_2e^{-\frac{\sqrt{\triangle_1}}{2}(x-\omega t)}}\Big]^2. \tag{3.5}$$

(2) When we choose $B \neq 0$ and $\triangle_1 = B^2+4A-4AC < 0$ and the triangular function solutions will be

$$u(x,t)=\frac{\varepsilon bw\triangle_1}{2a}+\frac{3B^2bw\triangle_1}{2a}\Big[\frac{ic_1\cos\frac{\sqrt{-\triangle_1}}{2}(x-\omega t)-c_2\sin\frac{\sqrt{-\triangle_1}}{2}(x-\omega t)}{ic_1\sin\frac{\sqrt{-\triangle_1}}{2}(x-\omega t)+c_2\cos\frac{\sqrt{-\triangle_1}}{2}(x-\omega t)}\Big]^2. \tag{3.6}$$

(3) If we choose $B=0$ and $\triangle_2 = A(C-1) \geq 0$, then the triangular function solutions are

$$u(x,t)=-\frac{2\lambda bw\triangle_2}{a}-\frac{6bw\triangle_2}{a}\Big[\frac{c_1\cos(\sqrt{\triangle_2}(x-\omega t))+c_2\sin(\sqrt{\triangle_2}(x-\omega t))}{c_1\sin(\sqrt{\triangle_2}(x-\omega t))-c_2\cos(\sqrt{\triangle_2}(x-\omega t))}\Big]^2. \tag{3.7}$$

(4) Again, when we choose $B=0$ and $\triangle_2 = A(C-1) < 0$, then hyperbolic function solutions are

$$u(x,t)=-\frac{2\lambda bw\triangle_2}{a}+\frac{6bw\triangle_2}{a}\Big[\frac{ic_1\cosh(\sqrt{-\triangle_2}(x-\omega t))-c_2\sinh(\sqrt{-\triangle_2}(x-\omega t))}{ic_1\sinh(\sqrt{-\triangle_2}(x-\omega t))-c_2\cosh(\sqrt{-\triangle_2}(x-\omega t))}\Big]^2. \tag{3.8}$$

Where A, B, C and c_1, c_2 are real parameters, a, b are positive constants. ε equals to 1 or 3, so is λ. And if ε equals to 1, we should choose $w=(b\triangle_1+1)^{-1}$, if ε equals to 3, then w is $(1-b\triangle_1)^{-1}$. Similarly, if λ is equal to 1 or 3, w is $(1-4b\triangle_2)^{-1}$ or $(1+4b\triangle_2)^{-1}$ respectively.

Eq. (3.5) can be rewritten at $c_1=-c_2$, as follows:

$$u(x,t)=\frac{\varepsilon bw\triangle_1}{2a}-\frac{3B^2bw\triangle_1}{2a}\tanh^2\frac{\sqrt{\triangle_1}}{2}(x-\omega t), \tag{3.9}$$

where $\triangle_1, w, A, B, C$ and a, b, ε are denoted by (3.5). Eq. (3.9) is the hyperbolic function solution of the RLW Eq. (3.9). Because of $\tanh^2 y = 1-\text{sech}^2 y$, Eq. (3.9) becomes

$$u(x,t)=\frac{bw\triangle_1}{2a}(\varepsilon-3B^2)+\frac{3B^2bw\triangle_1}{2a}\text{sech}^2\frac{\sqrt{\triangle_1}}{2}(x-\omega t), \tag{3.10}$$

where $\triangle_1, w, A, B, C$ and a, b, ε are denoted by (3.5). Comparing our hyperbolic type solution (3.10) and Kabir's results [12] (Eq. (3.6)), then it can be seen that the results are similar very much.

Again, Eq. (3.5) becomes at $c_1 = c_2$

$$u(x,t) = \frac{\varepsilon bw\triangle_1}{2a} - \frac{3B^2bw\triangle_1}{2a}\coth^2\frac{\sqrt{\triangle_1}}{2}(x-\omega t), \tag{3.11}$$

where $\triangle_1, w, A, B, C$ and a, b, ε are denoted by (3.5). Eq. (3.11) is similar with Eq. (3.7) in [12] after some simplifications.

3.2 Symmetric regularized long wave (SRLW) equation

Finally, Consider the following symmetric regularized long wave (SRLW) equation

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \tag{3.12}$$

which arises in severed physical applications including ion sound waves in plasma. Analogously, we introduce the variable $\xi = x - wt$, and make the transformation $u(x,t) = u(\xi)$, to reduce Eq. (3.12) to the ODE

$$w^2u'' + u'' - wuu'' - wu'^2 + w^2u'''' = 0. \tag{3.13}$$

Integrating Eq. (3.13) once with respect to ξ and setting the integration constant as zero yields

$$w^2u' + u' - wuu' + w^2u''' = 0. \tag{3.14}$$

Balancing the highest order nonlinear term uu' and the highest order partial derivative u''', we get

$$l + 3 = 2l + 1,$$

hence $l = 2$. So we can suppose that Eq. (3.14) has the following ansatz:

$$u(\xi) = m + nF(\xi) + kF^2(\xi), \tag{3.15}$$

where m, n, k are constants and need to be determined, $F(\xi)$ express the solution of Eq. (2.4)). Substituting Eqs. (3.15) and (2.5), along with (2.4) into Eq. (3.14) and using Maple yields a system of equations of $(G'/G)^i$, setting the coefficients of $(G'/G)^i$ $(i = 0, 1, 2, \cdots, 5)$ in the obtained system of equations to zero, we can deduce the following set of algebraic polynomials with the respect unknowns m, n, k namely:

$$2wk^2 - 2wk^2C - 24w^2k + 24w^2kC^3 - 72w^2kC^2 + 72w^2kC = 0,$$

$$\begin{aligned}&-2wk^2B + 54w^2kB - 3wnkC - 18w^2nC^2 + 18w^2nC - 108w^2kBC + 6w^2nC^3\\&+3wnk + 54w^2kBC^2 - 6w^2n = 0,\end{aligned}$$

$$\begin{aligned}&40w^2kA - 80w^2kAC - 2wk^2A - 3wnkB + 40w^2kAC^2 - 2w^2k + 2w^2kC - 24w^2nBC + 2wmk\\&-2k - wn^2C - 2wmkC + 38w^2kB^2C + wn^2 + 12w^2nB - 38w^2kB^2 + 2kC + 12w^2nBC^2 = 0,\end{aligned}$$

$$\begin{aligned}
&-52w^2kAB+2w^2kB-n+8w^2nA-wmnC-w^2n+nC-2wmkB+w^2nC+wmn-7w^2nB^2\\
&+52w^2kABC-3wnkA-wn^2B-16w^2nAC+8w^2kB^3+8w^2nAC^2+7w^2nB^2C+2kB=0,\\
&w^2nB+2w^2kA+w^2nB^3-wmnB+nB-wn^2A-8w^2nBA+16w^2kCA^2+14w^2kAB^2\\
&+2kA-16w^2kA^2+8w^2nBCA-2wmkA=0,\\
&nA+w^2nA-2w^2nA^2+w^2nB^2A-wmnA+6w^2kBA^2+2w^2nCA^2=0.
\end{aligned}$$

Solving the set of algebraic equations by use of Maple, we get the following results:

$$\Big\{m=\frac{(w^2+w^2B^2+1-8w^2A+8w^2AC)}{w},\ \ n=12wB(C-1),\ \ k=12w(-2C+1+C^2)\Big\}.$$

Therefore, substitute the above set in (3.15), we get

$$\begin{aligned}
u(\xi)=&\frac{(w^2+w^2B^2+1-8w^2A+8w^2AC)}{w}+12wB(C-1)F(\xi)\\
&+12w(-2C+1+C^2)F^2(\xi).
\end{aligned}\tag{3.16}$$

Substituting the general solutions (2.6)-(2.8) into Eq. (3.16), we obtain three types of traveling wave solutions of Eq. (3.12) in view of the positive,negative or zero of B and $\triangle$:

(1) When we choose $B\neq 0$ and $\triangle=B^2+4A-4AC\geq 0$, then the exponential function solutions can be found as

$$u(x,t)=\frac{w^2+1-2w^2\triangle}{w}+3wB^2\triangle\Big[\frac{c_1e^{\frac{\sqrt{\triangle}}{2}(x-\omega t)}+c_2e^{-\frac{\sqrt{\triangle}}{2}(x-\omega t)}}{c_1e^{\frac{\sqrt{\triangle}}{2}(x-\omega t)}-c_2e^{-\frac{\sqrt{\triangle}}{2}(x-\omega t)}}\Big]^2.\tag{3.17}$$

(2) When we choose $B\neq 0$ and $\triangle=B^2+4A-4AC<0$ and the triangular function solutions will be

$$u(x,t)=\frac{w^2+1-2w^2\triangle}{w}-3wB^2\triangle\Big[\frac{ic_1\cos\frac{\sqrt{-\triangle}}{2}(x-\omega t)-c_2\sin\frac{\sqrt{-\triangle}}{2}(x-\omega t)}{ic_1\sin\frac{\sqrt{-\triangle}}{2}(x-\omega t)+c_2\cos\frac{\sqrt{-\triangle}}{2}(x-\omega t)}\Big]^2.\tag{3.18}$$

(3) If we choose $B=0$ and $\triangle=A(C-1)\geq 0$, then the triangular function solutions are

$$u(x,t)=\frac{w^2+1+8w^2\triangle}{w}+12w\triangle\Big[\frac{c_1\cos(\sqrt{\triangle}(x-\omega t))+c_2\sin(\sqrt{\triangle}(x-\omega t))}{c_1\sin(\sqrt{\triangle}(x-\omega t))-c_2\cos(\sqrt{\triangle}(x-\omega t))}\Big]^2.\tag{3.19}$$

(4) Again, when we choose $B=0$ and $\triangle=A(C-1)<0$, then hyperbolic function solutions are

$$u(x,t)=\frac{w^2+1+8w^2\triangle}{w}-12w\triangle\Big[\frac{ic_1\cosh(\sqrt{-\triangle}(x-\omega t))-c_2\sinh(\sqrt{-\triangle}(x-\omega t))}{ic_1\sinh(\sqrt{-\triangle}(x-\omega t))-c_2\cosh(\sqrt{-\triangle}(x-\omega t))}\Big]^2.\tag{3.20}$$

Where A, B, C, w and c_1, c_2 are real parameters.

Eq. (3.17) can be rewritten at $c_1 = -c_2$, as follows:

$$u(x,t) = \frac{w^2 + 1 - 2w^2\triangle}{w} + 3wB^2\triangle\tanh^2\frac{\sqrt{\triangle}}{2}(x - \omega t), \tag{3.21}$$

where $\triangle, w, A, B$ and C are denoted by (3.17). Eq. (3.21) is the hyperbolic function solution of the SRLW Eq. (3.12).

Substituting $\tanh^2 y = 1 - \mathrm{sech}^2 y$, and $\triangle = c_2 > 0$, $w = -c$, $B^2 = 1$, into Eq. (3.21) yields

$$u(x,t) = 3cc_2\mathrm{sech}^2\frac{\sqrt{c_2}}{2}(x + ct) - \frac{c^2 + 1 + c^2c_2}{c}. \tag{3.22}$$

Comparing our hyperbolic type solution (3.22) and Xu's results [13] (Eq. (3.10)), we found that the results are the same.

4 Conclusions

In this work, we have presented an improved (G'/G)-expansion method and applied it to obtain new traveling wave solutions of the RLW equation and SRLW equation. In contrast to other (G'/G)-expansion method, some benefits are available for this method.

First, all the nonlinear PDEs which can be solved by other (G'/G)-expansion method can be solved easily by this method.We have successfully obtained many more new exact traveling wave solutions. To our knowledge, these solutions have not been reported in former literature.

Second, if we used the special value of parameters c_1, c_2 and A, B, C, we can obtain some traveling wave (19,20,31) which have been found by Kabir [12] and Xu [13].

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