

Development of Multi-hierarchy Simulation Model for Studies of Magnetic Reconnection

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Abstract. The multi-hierarchy simulation model for magnetic reconnection is developed, where both micro and macro hierarchies are expressed consistently and simultaneously. Two hierarchies are connected smoothly by shake-hand scheme. As a numerical test, propagation of one-dimensional Alfvén wave is examined using the multi-hierarchy simulation model. It is found that waves smoothly pass through from macro to micro hierarchies and *vice versa*.

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Key words: Multi-hierarchy, magnetic reconnection, MHD, particle-in-cell.

1 Introduction

Magnetic reconnection is a fundamental process to lead to the fast energy release from magnetic field to plasmas. For instance, solar flares [1,2], Earth magnetic substorms [3], and tokamak disruptions [4] are widely believed to be triggered by magnetic reconnection. Even though magnetic reconnection causes macroscopic phenomenon that global field topology changes, such high-temperature and low-density plasmas are collisionless, and frozen-in condition is satisfied macroscopically. Hence, occurrence of magnetic reconnection requires microscopic processes which break the frozen-in constraint. Namely, magnetic reconnection is a phenomenon bridging across different hierarchies, and thus the full understanding of magnetic reconnection needs a multi-hierarchy model which can deal with both microscopic and macroscopic physics consistently and simultaneously [5].

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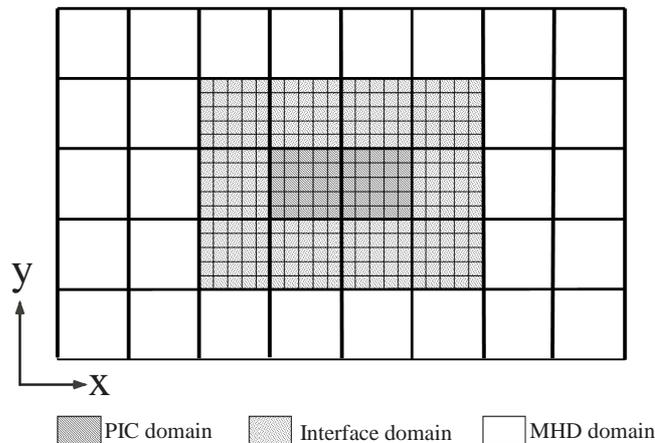


Figure 1: Schematic diagram of multi-hierarchy model. Simulation domain is divided into three domains; PIC, MHD, and Interface domains.

In this paper, we propose a multi-hierarchy simulation model which we are developing. In Section 2, we explain our multi-hierarchy model. Numerical methods to interlock between different hierarchies are shown. In Section 3, we examine our model. It is observed that one-dimensional Alfvén wave smoothly propagates in a multi-hierarchy simulation box. In Section 4, we discuss problems on the multi-hierarchy simulation demonstrated in Section 3. Section 5 gives a summary of our work.

2 Multi-hierarchy simulation scheme

In this section, we describe algorithm for multi-hierarchy simulation of magnetic reconnection, i.e., two hierarchies which make up our model, how to interlock two hierarchies smoothly, unit transformation, and data exchange.

2.1 Two hierarchies

Our multi-hierarchy model is based on the domain division method, and thus is composed of two hierarchies: micro hierarchy and macro hierarchy. The neighborhood of reconnection points is micro hierarchy, where microscopic kinetic effects play crucial roles, and frozen-in condition is violated. Dynamics in this system are solved by explicit electromagnetic particle-in-cell (PIC) simulation [6–9]. Let us give the name ‘PIC domain’ to this domain. On the other hand, the surrounding of PIC domain is described by magnetohydrodynamic (MHD) simulation [10]. In ‘MHD domain’, ideal MHD equations are applicable, since non-ideal effects leading to the generation of electric resistivity and viscosity are assumed to be generated by microscopic physics in the vicinity of reconnection points [11–13].

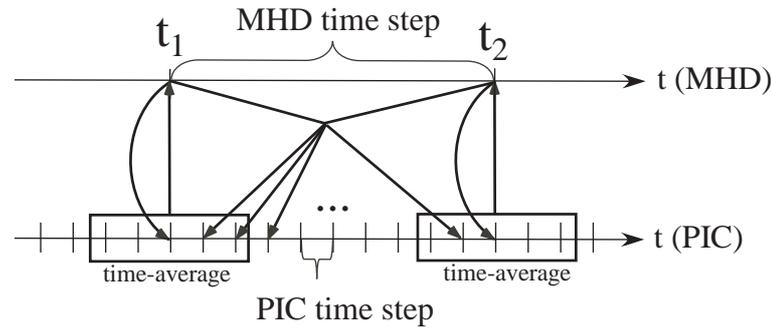


Figure 2: Time-flow of multi-hierarchy model. MHD and PIC time steps differ.

2.2 Space: Shake-hand scheme

Fig. 1 is the schematic diagram of our multi-hierarchy simulation model, where system is expressed as two-dimensional mesh, however, actual simulation would be performed as three-dimensional system. With this figure, we show the scheme to interlock between MHD and PIC domains. The fine cells are used for PIC model, while the coarse ones are for MHD model. Reconnection points are located in PIC domain. The neighborhood of reconnection points, where microscopic physics plays an important role, is solved by PIC model, and the surrounding of the PIC domain is described by MHD simulation model. Between PIC and MHD domains, we insert 'Interface domain' which has finite width [14]. MHD and PIC domain can be smoothly interlocked with 'shake-hand scheme'. Interface domain is solved by both MHD and PIC models. Let us consider a certain field quantity $Q(x, y, z)$ (for instance, Q is magnetic field). In Interface domain, quantities Q_{MHD} and Q_{PIC} are obtained, where Q_{MHD} and Q_{PIC} are the values of Q calculated from MHD and PIC models, respectively. Then, Q in Interface domain is given as the value interpolated of Q_{MHD} and Q_{PIC} as follows,

$$Q = aQ_{\text{MHD}} + (1-a)Q_{\text{PIC}}, \quad (2.1)$$

where parameter a is a function of x , y , and z .

2.3 Time: Multi-time scale scheme

Next, we show simulation time-flow of our multi-hierarchy model in Fig. 2. We employ multi-time scale scheme, where MHD and PIC domains have different time steps each other. Large time steps are for MHD model, and small ones are for PIC model. For advancing time from t_1 to t_2 , PIC model receives interpolation values of data at t_1 and at t_2 from MHD model every step. On the other hand, at t_1 , MHD model gets PIC data averaged over several steps around t_1 .

The following procedure indicates how time advances from t_1 to t_2 . (1) At $t = t_1$, physical quantities of MHD and PIC exist. (2) MHD model sends MHD information at t_1

Table 1: Normalization constants in MHD and PIC equations. Here, Δ , B_0 , and ρ_0 are arbitrary, c is the speed of light, ω_{ce} is the electron gyrofrequency, v_A is the Alfvén speed ($v_A = B_0 \rho_0^{-1/2}$), m_e^{SP} is the electron super-particle mass, and q_e^{SP} is the electron super-particle charge.

Quantity	MHD normalization	PIC normalization
Length	Δ	c / ω_{ce}
Velocity	v_A	c
Time	Δ / v_A	$1 / \omega_{ce}$
Magnetic field	B_0	$m_e^{SP} c \omega_{ce} / q_e^{SP}$
Electric field	-	$m_e^{SP} c \omega_{ce} / q_e^{SP}$
Mass	-	m_e^{SP}
Charge	-	q_e^{SP}
Mass density	ρ_0	-
Pressure	$\rho_0 v_A^2$	-

to PIC model. (3) PIC model advances to $t = t_1 + \delta t$, where δt is time which corresponds to several PIC steps. (4) PIC information averaged over the period from $t_1 - \delta t$ to $t_1 + \delta t$ is sent to MHD model. (5) MHD model goes head one step and reaches t_2 . (6) PIC model receives MHD data at t_1 and at t_2 every step from t_1 to t_2 . (7) PIC information from t_1 to $t_1 + \delta t$ which were obtained at (3) is removed. (8) PIC model advances to t_2 .

2.4 Unit transformation and data exchange

Spatial and temporal sizes treated in MHD model are much different from ones in PIC model, and hence not only physical quantities but also normalization constants are different each other. Table 1 represents physical quantities and normalization constants in MHD and PIC models. For instance, velocities in MHD model are normalized to the Alfvén speed v_A , while ones in PIC model are normalized to the speed of light c .

Thus, MHD-PIC interlocked model requires to transform unit. We thereby derive the constitution of unit transformation. First of all, it is supposed that normalization constant of magnetic field in MHD model equals to that in PIC model;

$$B_0 = \frac{m_e^{SP} c \omega_{ce}}{q_e^{SP}}, \quad (2.2)$$

where B_0 is arbitrary magnetic field, m_e^{SP} is the electron super-particle mass, ω_{ce} is the electron gyrofrequency, and q_e^{SP} is the electron super-particle charge. Second, let us define

$$\Delta = \alpha (c / \omega_{ce}), \quad (2.3)$$

which means that MHD unit length is α times PIC unit length. We can determine this relation artificially. By virtue of Eqs. (2.2) and (2.3), unit-transformation relations of other constants are given uniquely.

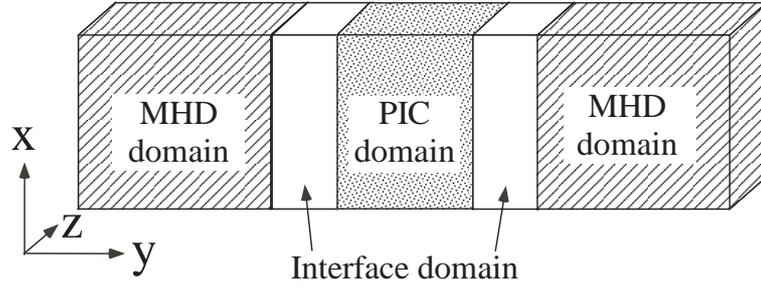


Figure 3: The schematic diagram of simulation box. Periodic boundary condition for all directions is satisfied as whole box.

PIC model has positions and velocities of particles, and electromagnetic field as physical quantities. On the other hand, quantities which treated by MHD model are fluid velocity, magnetic field (\mathbf{B}), mass density, and pressure. PIC model obtains fluid velocity, pressure and mass density with statistics. Then, PIC model sends fluid velocity, magnetic field, mass density, and pressure to MHD model. Now, it is assumed that frozen-in condition is satisfied in MHD model, thus MHD model can calculate electric field from velocity and magnetic field. Furthermore, MHD model obtains thermal velocity and number density using mass density and pressure. MHD model gives fluid velocity, magnetic field, electric field, number density (n), and thermal velocity to PIC model. PIC model produces shifted Maxwellian velocity distribution based on number density, fluid and thermal velocities. It would suitable that in PIC model, electron particle velocities v_e are changed into $v_e - (\nabla \times \mathbf{B})/n$, since the difference between ion and electron velocities produces electric current.

3 Numerical test of multi-hierarchy model

In order to examine our multi-hierarchy model, we perform simulation of propagation of one-dimensional linear Alfvén wave in the box as shown in Fig. 3. The simulation box size is $16(c/\omega_{ce}) \times 256(c/\omega_{ce}) \times 4(c/\omega_{ce})$. Here, α in Eq. (2.3) was taken to be 1.0. Also, 100 PIC time steps correspond to 1 MHD time step. PIC domain is located in the center of the simulation box ($96 < \hat{y} < 160$, $\hat{y} = y/(c/\omega_{ce})$), and MHD domains are in both sides of PIC domain ($0 < \hat{y} < 64$, $192 < \hat{y} < 256$). There exists Interface domain between MHD and PIC domains ($64 < \hat{y} < 96$, $160 < \hat{y} < 192$). As interpolation of Q_{MHD} and Q_{PIC} , the relations

$$Q = \left(\frac{96 - \hat{y}}{32} \right) Q_{\text{MHD}} + \left(1 - \frac{96 - \hat{y}}{32} \right) Q_{\text{PIC}} \quad \text{for } 64 < \hat{y} < 96, \quad (3.1)$$

$$Q = \left(\frac{\hat{y} - 160}{32} \right) Q_{\text{MHD}} + \left(1 - \frac{\hat{y} - 160}{32} \right) Q_{\text{PIC}} \quad \text{for } 160 < \hat{y} < 192, \quad (3.2)$$

are used. The ion-to-electron mass ratio is $m_i/m_e = 100$, and the ratio of the plasma frequency to the electron gyrofrequency is $\omega_{pe}/\omega_{ce} = 1.0$. The following boundary con-

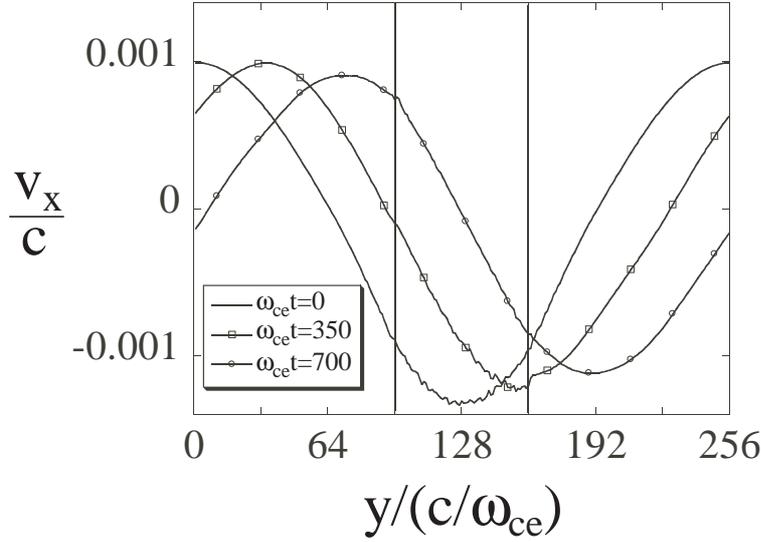


Figure 4: Spatial profiles of fluid velocity v_x at $\omega_{pe}t=0, 350,$ and 700 . Wave propagates in the right direction.

dition is taken. The right-side of right MHD domain is connected to the left-side of left MHD domain as a periodic boundary condition. The x and z directions perpendicular to the wave normal are also periodic. The external magnetic field is taken to be in the y direction; $\mathbf{B} = (0, B_{y0}, 0)$, and Alfvén wave propagates in the y direction.

Fig. 4 displays spatial profiles of x component of fluid velocity v_x at the various times. The amplitude of magnetic field is $\delta B/B_{y0}=0.01$, and wavelength is $\lambda = 256c/\omega_{ce}$. We can see small noises only in PIC domain. They are caused by thermal fluctuation. Alfvén wave smoothly propagates in the right direction through PIC domain. The propagation speed is observed to be v_A .

4 Discussion

In order to make linear Alfvén wave be clearly observed, it is necessary that thermal velocity is of the order of or less than fluid velocity in PIC simulation. From $\delta B/B_{y0}=0.01$ and $\lambda=256c/\omega_{ce}$, electron fluid velocity is found as $v_e/c \simeq 0.001$. Therefore, in simulation shown in Section 3, electron thermal velocity needs to satisfy

$$v_{Te}/c \lesssim 0.001. \quad (4.1)$$

Eq. (4.1) leads to the Debye length as

$$\frac{\lambda_{De}}{c/\omega_{ce}} \lesssim 0.001. \quad (4.2)$$

On explicit electromagnetic PIC method, the condition for grid spacing Δ_g

$$\frac{\Delta_g}{\lambda_{De}} < 1 \quad (4.3)$$

is necessary. This condition requires that $\Delta_g \lesssim 0.001c/\omega_{ce}$. In this paper, however, because of the limitation of simulation time and memory size, we reluctantly used $\Delta_g = c/\omega_{ce}$. We hence did not perform simulation for extremely long time. However, until $\omega_{pe}t \simeq 700$, large numerical instabilities did not be produced. We believe that our simulation results indicate correct physical phenomena at least until $\omega_{ce}t = 700$.

5 Summary

For magnetic reconnection studies, we have been developing multi-hierarchy simulation model, where both microscopic and macroscopic physics are solved consistently and simultaneously. Microscopic and macroscopic dynamics are expressed by explicit electromagnetic PIC and MHD models, respectively. This model is based on the domain division method, and simulation box is divided into PIC, MHD, and Interface domains. Through Interface domain, PIC and MHD domains are interlocked with each other. Indeed, our simulation has demonstrated that one-dimensional linear Alfvén wave smoothly propagates in our multi-hierarchy model.

As future work, it would be interesting to carry out simulations on other waves such as magnetosonic wave to verify our model. After verifying, we would study magnetic reconnection using our multi-hierarchy model, and furthermore would apply our simulation to Earth magnetosphere, *etc.*

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