

Combination of Implicit Integration and Collision Response for Cloth Simulation

Hongyan Liu ^a, Yueqi Zhong ^{a,b,*}, Shanyuan Wang ^a

^a*College of Textiles Donghua University, Shanghai 201620, China*

^b*Key Laboratory of Ministry of Education on Textile Fabric Technology, Donghua University
Shanghai 201620, China*

Abstract

This paper implements efficient techniques for cloth simulation in the area of cloth model, numerical integration, collision detection and response. The whole procedure combines the accurate bending model with implicit integration and collision response. Collision response remains a serious difficulty in cloth simulation, especially with implicit integration. We propose a general scheme that integrates collision response into implicit models recovering most of the nice draping movements of the cloth. Collision detection between human body and cloth is accelerated by Axis Aligned Bounding Box (AABB). A cloth draped on a human model is tested for the combined techniques we put forward. Experimental results reveal that this procedure is suitable for the accurate cloth simulation which preserves folds and wrinkles. The scheme that combines implicit integration and two-stage collision response is efficient for cloth simulation.

Keywords: Implicit Integration; Collision Response; Combination; Cloth Simulation

1 Introduction

The technology involved in virtual sewing and draping is generally regarded as a physically-based method. Among which, continuum approach [1-6] and particle-based approach [7-12] are often referred. Feynman's model [6] for generating the appearance of cloth is one of the earliest works. By regarding the cloth object as an elastic plate, the final drape was computed by finding the minimum value of the energy equation. In the work of Terzopoulos et al. [1], the cloth object was also represented as an elastic object. However this method suffers from instability problems that decrease overall performance. The first milestone of non-continuum approach was set by Breen et al. [8] in their famous particle-based model to predict the draping behavior of woven cloth. The cloth object was represented as an interlaced particle system. The method was devised to achieve the final equilibrium state of specific materials. Energy minimization was also employed to determine the equilibrium position. The drawback of this method is that it cannot produce

*Corresponding author.

Email address: zhyq@dhu.edu.cn (Yueqi Zhong).

transitions between the initial state and the final equilibrium. Years later, House et al. [13] extended this model with force-based techniques, which was very similar with the most popular mass-spring model proposed by Provot [12] in 1995. Actually, the major contribution of Provot is his attempt to describe the rigid behavior of cloth with a position adjustment method to overcome the super-elasticity, which is a hot topic in many physically-based methods [14-24]. Eberhardt et al. [9] expanded the particle-based model by incorporating hysteresis and creases to create a dynamic simulation method based on a Lagrangian formulation. Baraff and Witkin [11] used a continuum approach on each triangle for in-plane deformation and the angle between adjacent triangles to measure out-of-plane deformation, their implicit integration scheme was the most advocated method in recent years.

Collision handling remains a serious difficulty in cloth simulation, especially with implicit integration. We propose a general scheme that integrates collision response into implicit Euler models instead of implicit Midpoint method [25] recovering most of the nice draping movements of the cloth.

2 Algorithms

2.1 Physical Model

The present work employs a mass-spring system [12] for garments based on triangular mesh. Each vertex is considered as a mass and the triangle edges are considered as the springs. The stretching and shearing resistance is enforced by applying the spring force along each edge in Eq. (1).

$$\mathbf{f} = \mathbf{f}_i + \mathbf{d}_i = -k \frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_i} \mathbf{C}(\mathbf{x}) - k_d \frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_i} \mathbf{C}(\mathbf{x}) \quad (1)$$

where \mathbf{f}_i and \mathbf{d}_i are the elastic force and damping force applied onto i th mass. \mathbf{C} is the conditional function. By defining $\mathbf{C} = |\mathbf{x}_{ij}| - L$, we have:

$$\mathbf{f}_i = \begin{cases} k_s (|\mathbf{x}_{ij}| - L) \frac{\mathbf{X}_{ij}}{|\mathbf{x}_{ij}|} & : |\mathbf{x}_{ij}| \geq L \\ 0 & : |\mathbf{x}_{ij}| < L \end{cases}$$

where $\mathbf{x}_{ij} = \mathbf{x}_j - \mathbf{x}_i$, L is the rest length between i th and j th masses.

The bending resistance is enforced by resisting the rotation among two adjacent triangles who shares the same edge as illuminated in Fig. 1. The bending force is implemented as in the work of Bridson et al. [26] in Eq. (2).

$$\begin{aligned} \mathbf{f} &= \mathbf{f}_i^e + \mathbf{f}_i^d = k_e \frac{|\mathbf{E}|^2}{|\mathbf{N}_1| + |\mathbf{N}_2|} (\sin(\theta/2)) \mathbf{u}_i - k_d |\mathbf{E}| (\mathbf{u} \otimes \mathbf{u}) \mathbf{v} \\ \mathbf{u}_1 &= |\mathbf{E}| \frac{\mathbf{N}_1}{|\mathbf{N}_1|^2}, \quad \mathbf{u}_3 = \frac{(\mathbf{x}_1 - \mathbf{x}_4) \cdot \mathbf{E}}{|\mathbf{E}|} \frac{\mathbf{N}_1}{|\mathbf{N}_1|^2} + \frac{(\mathbf{x}_2 - \mathbf{x}_4) \cdot \mathbf{E}}{|\mathbf{E}|} \frac{\mathbf{N}_2}{|\mathbf{N}_2|^2} \\ \mathbf{u}_2 &= |\mathbf{E}| \frac{\mathbf{N}_2}{|\mathbf{N}_2|^2}, \quad \mathbf{u}_4 = \frac{(\mathbf{x}_1 - \mathbf{x}_3) \cdot \mathbf{E}}{|\mathbf{E}|} \frac{\mathbf{N}_1}{|\mathbf{N}_1|^2} - \frac{(\mathbf{x}_2 - \mathbf{x}_3) \cdot \mathbf{E}}{|\mathbf{E}|} \frac{\mathbf{N}_2}{|\mathbf{N}_2|^2} \end{aligned} \quad (2)$$

where $i=1, 2, 3, 4$, $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4)$ represent motion mode that changes the dihedral angle but does not cause any in-plane deformation or rigid body motion. \mathbf{f}_i^e and \mathbf{f}_i^d denotes elastic and damping bending forces respectively. k_e and k_d is the elastic bending stiffness and damping coefficient respectively.

$$\begin{aligned} \mathbf{N}_1 &= (\mathbf{x}_1 - \mathbf{x}_3) \times (\mathbf{x}_1 - \mathbf{x}_4) \\ \mathbf{N}_2 &= (\mathbf{x}_2 - \mathbf{x}_4) \times (\mathbf{x}_2 - \mathbf{x}_3) \end{aligned}$$

\mathbf{N}_1 and \mathbf{N}_2 is the area weighted normal, $\mathbf{E} = \mathbf{x}_4 - \mathbf{x}_3$ is the common edge.

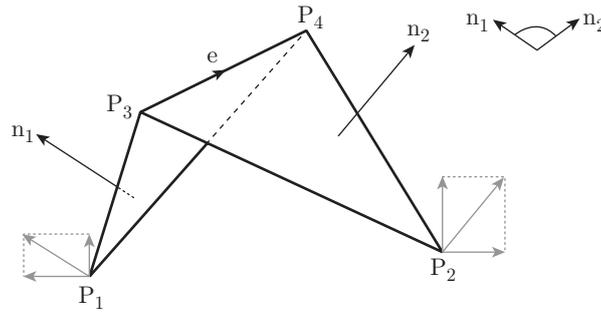


Fig. 1: Model of bending resistance

2.2 Implicit Integration

The numerical errors of standard explicit integration schemes accumulate thus creating numerical instability, unless the time step is small enough. Implicit methods do not exhibit this behavior and allow large time steps. Baraff and Witkin [11] made a breakthrough in cloth simulation by introducing implicit Euler methods in a particle system. For a numerical stable garment model, implicit integration methods [11] are adopted for solving differential equation systems which are known to remain stable for stiff problems. The MPCG (Modified Preconditioned Conjugate Gradient) method is taken for solving the linear equation system in Eq. (3).

$$\begin{aligned} \begin{pmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{v} \end{pmatrix} &= h \begin{pmatrix} \mathbf{v}_0 + \Delta \mathbf{v} \\ \mathbf{M}^{-1} \mathbf{f}(\mathbf{x}_0 + \Delta \mathbf{x}, \mathbf{v}_0 + \Delta \mathbf{v}) \end{pmatrix} \\ \mathbf{f}(\mathbf{x}_0 + \Delta \mathbf{x}, \mathbf{v}_0 + \Delta \mathbf{v}) &= \mathbf{f}_0 + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \Delta \mathbf{v} \\ \begin{pmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{v} \end{pmatrix} &= h \begin{pmatrix} \mathbf{v}_0 + \Delta \mathbf{v} \\ \mathbf{M}^{-1} \left(\mathbf{f}_0 + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \Delta \mathbf{v} \right) \end{pmatrix} \\ \Delta \mathbf{v} &= h \mathbf{M}^{-1} \left(\mathbf{f}_0 \frac{\partial \mathbf{f}}{\partial \mathbf{x}} h(\mathbf{v}_0 + \Delta \mathbf{v}) + \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \Delta \mathbf{v} \right) \end{aligned} \quad (3)$$

To be consistent with the collision handling method the momentum-conserving corrective impulses are applied to particles for strain limiting [16] which allows to have correct positions and corresponding velocities. The state of the cloth after the limiting strain is the input for the collision algorithm.

2.3 Collision Detection and Response

Collision detection is known as the major bottleneck in cloth simulation. The intersection is located via AABB (Axis Aligned Bounding Box) for accelerating detection of collision between garment and human body rather than OBB (Oriented Bounding Box) because that they are not much slower to test, but faster to build, and uses less storage than OBB trees [27]. The AABB binary tree is built top-down in the beginning of the simulation. The set is split by comparing the geometrical center of primitives with respect to a well-chosen partitioning plane. This process continues until each subset contains one element. Thus, an AABB tree for a set of n primitives has n leaves and $n-1$ internal nodes. We then split the set into a negative and positive subset corresponding to the respective half spaces of the plane. A primitive is classified as positive if the midpoint of its projection onto the axis is greater than a certain threshold, and negative otherwise. The deformable garment model is a triangle mesh in which the coordinates of the vertices change over time. AABB tree is updated using bottom-up when the garment is deformed after each time step. As stated by Bergen [27] refitting a AABB tree is much faster than rebuilding it.

2.4 Intersection Testing

The collision algorithm that we implemented is similar to what was proposed by Bridson et al. [16]. Thus the following is just a brief description. Given four points and their corresponding velocities, the geometric collision tests consist of determining the time when the four points are coplanar. If t is inside the current time step, a proximity test is performed using the position of the points at time t . Given four points $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$, their velocities $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$, and defining $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$, the time t when the four points are coplanar are the roots of the cubic Eq. (4) [28].

$$(\mathbf{x}_{21} + t\mathbf{v}_{21}) \times (\mathbf{x}_{31} + t\mathbf{v}_{31}) \cdot (\mathbf{x}_{41} + t\mathbf{v}_{41}) = 0 \quad (4)$$

Eq. (4) is a cubic equation:

$$a_3 t^3 + a_2 t^2 + a_1 t + a_0 = 0$$

where

$$\begin{aligned} a_3 &= f \cdot (b \times d) & a &= \mathbf{x}_{21} \\ a_2 &= e \cdot (b \times d) + f \cdot (b \times c + a \times d) & b &= \mathbf{v}_{21} \\ a_1 &= f \cdot (a \times c) + e \cdot (b \times c + a \times d) & c &= \mathbf{x}_{31} \\ a_0 &= e \cdot (a \times c) & d &= \mathbf{v}_{31} \\ & & e &= \mathbf{x}_{41} \\ & & f &= \mathbf{v}_{41} \end{aligned} \quad \text{and}$$

To solve the cubic equation the Cardano's formula described in [29] is adopted. Collision is detected by "point-triangle" and "edge-edge" proximity tests. If a point \mathbf{p} is closer to a triangle $\mathbf{t}_1 \mathbf{t}_2 \mathbf{t}_3$ than a certain threshold, the point \mathbf{p} is projected to the triangle plane and its barycentric coordinates are computed to test whether the projected point is inside the triangle for collision registration. The "edge-edge" proximity tests are implemented by the distance between segments.

The collision detection provides information about collision time and collision normal which is needed in the later collision handling process.

The collision algorithm adopted in this paper is similar to the one proposed by Bridson et al. [16]. Two stages are involved; we try to prevent collision from happening in the first stage, and the second stage deals with actual intersections. This two-stage approach eliminates almost all collision in the first stage, leaving only a few collisions for the second stage.

3 Experimental Results and Conclusion

In our implementation, a square cloth is draped onto a human model to validate our methods. All the experiments are conducted on a PC with a 1.61 GHz CPU and 2.00 GB physical memory.

The collisions between cloth and the avatar are detected as illustrated in Fig. 2. The triangles which are in the overlapped AABBs are tested further to check the collisions between the elements of mesh. The overlapped AABBs are demonstrated in Fig. 3 which accelerates collision detection. After the two-stage collision response the penetrations are resolved as in Fig. 4. The combination of implicit integration method and the two-stage collision response scheme is suited for efficient cloth simulation. The method proposed in this paper exhibits pretty good performance for fast or real-time cloth simulation. Further work needs to be conducted on complex garments and avatars.

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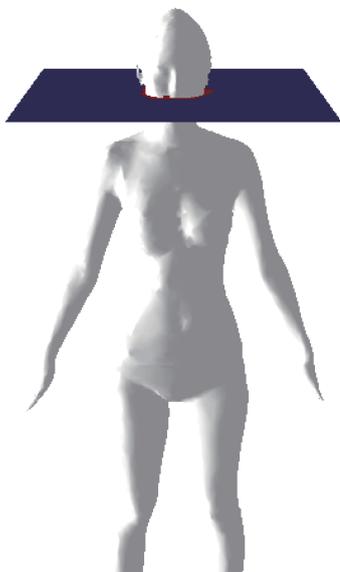


Fig. 2: Detected collision particles (red)

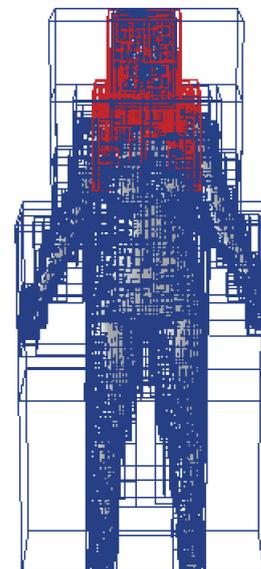


Fig. 3: AABB of square cloth (red) and human body (blue)



Fig. 4: A squared cloth draped onto human model without (left) and with (right) penetration resolution

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